

# FINDING UNDERLYING FACTORS USING THE INDEPENDENT COMPONENT ANALYSIS ON LABOUR MARKET – APPLICATION ON UNEMPLOYMENT RATE IN MONTHLY VARIATION

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## **Abstract**

*Independent Component Analysis ICA is “a method for finding underlying factors or components from multivariate (multidimensional) statistical data”. Considering that the specific of this method is “that it looks for components that are both statistically independent and Non-Gaussian, we try to apply ICA method on labour market data. Following the methodology presented by Hyvärinen, Karhunen, Oja (2001) on the problem “cashflow of several stores belonging to the same retail chain, trying to find fundamental factors common to all stores that affect the cash flow” we apply on analysing the unemployment rates, seasonally adjusted, in monthly variation at EU27 level between January 2000-September 2011. The data source is EUROSTAT, indicator [une\_rt\_m]: „Unemployment rate, monthly average, seasonally adjusted data, total (%), resulting 141 months/cases. Original mixture data are pre-processing in the stage of Pre-whitened using Principal Component Analysis PCA, with NIPALS algorithm and for ICA the FastICA Algorithm from STATISTICA 8.0 Software.*

**Keywords: unemployment rate, monthly variation, ICA, PCA**

**JEL classification: J6**

*„Most measured quantities are actually mixtures of other quantities..”<sup>1</sup>*

## **1. Introduction**

Starting from Hyvärinen, Karhunen, Oja financial applications<sup>2</sup> of Independent Component Analysis ICA as “a method for finding underlying factors or components from multivariate (multidimensional) statistical data”, considering that the specific of this method is “that it looks for components that are both statistically independent and nongaussian, we try to apply ICA method on labour market data.

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<sup>1</sup> J.V. Stone, (2005): *A Brief Introduction to Independent Component Analysis* in Encyclopedia of Statistics in Behavioral Science, Volume 2, pp. 907–912, Editors Brian S. Everitt & David C. Howell, John Wiley & Sons, Ltd, Chichester, 2005 ISBN 978-0-470-86080-9

<sup>2</sup> K. Kiviluoto și E. Oja, Independent component analysis for parallel financial time series. În *Proc. ICONIP'98, vol.2, pg. 895-898, Tokio, Japan, 1998.*

Following the methodology presented<sup>3</sup> on the problem” cashflow of several stores belonging to the same retail chain, trying to find fundamental factors common to all stores that affect the cash flow” we apply on analysing the unemployment rate, seasonally adjusted, in monthly variation at EU27 level between January 2000-September 2011. The data source is EUROSTAT, indicator [une\_rt\_m]: „Unemployment rate, monthly average, Seasonally adjusted data, total (%), resulting 141 months/cases.

ICA is based on the idea that independence is a stronger property than uncorrelatedness [PCA]. Uncorrelatedness in itself is not enough to separate the components”, ”PCA or factor analysis cannot separate the signals: they gave components that are uncorrelated”<sup>4</sup> Among the applications of the ICA there are already some on econometrics – parallel time series financial data<sup>5</sup>.

## 2. Data treatment as signal from informational perspective

According with the ICA theory the working unit is a signal. If “any signal represents form the mathematical point of view a time function”<sup>6</sup> then we shall point some specific characteristics under this assumption:

- On the labour market there are studied behaviours of the individual during the economic activity span. If there are used micro data organised in longitudinal data base we can tell that we increase the chances to model better the reality. When reference is to the individual the variables could be compared with “**analogical signals**”: the individual lives as continuous function with emitted continuous values in  $\mathbb{R}$ <sup>7</sup>

$$f : \mathcal{R} \rightarrow \mathcal{R}$$

$$f : [\mathcal{R}] \rightarrow [\mathcal{R}] \quad (1)$$

- Data treatment as signal from informational perspective allow the conversion analogue in digital or the conversion from continuum in discreet<sup>8</sup> through precise procedures that combines operations like: discretization, sampling, quantification, coding, etc. In the technical literature those operations could be described as functions of time (see **Fig. 1**)
- Signals transmits amplitudes and „any signal could be decompose in sum of the sinusoidal signals –subject of the **Fourier Analysis**”<sup>9</sup>.
- Sinusoidal signals are orthogonal and are the only ones from nature able to propagate through linear systems without to be distorted (is changing only the amplitude and phase). Sinusoidal signals contains extremely small information:

$$x(t) = x(A, T, t) \quad (2)$$

$$x(t) = A \sin(\omega * t)$$

$$f = \frac{1}{T}$$

$$\varphi = \omega * t$$

<sup>3</sup> Hyvärinen, Karhunen, Oja, Independent Component Analysis, 2001 John Wiley & Sons, pg.441.

<sup>4</sup> Hyvärinen, Karhunen, Oja, Independent Component Analysis, 2001 John Wiley & Sons, pg.7.

<sup>5</sup> Kiviluoto K. and E. Oja. Independent component analysis for parallel financial time series. In *Proc. ICONIP'98*, volume 2, pages 895-898, Tokyo, Japan, 1998.

<sup>6</sup> Ioan P. Mihu, Despre semnale și sisteme din volumul Procesarea numerică a Semnalelor, pg.2

<sup>7</sup> Laurențiu Frangu – Introducere în Inginerie Electronică și Telecomunicații, 2008, pg. 30

<sup>8</sup> Laurențiu Frangu – Introducere în Inginerie Electronică și Telecomunicații, 2008

<sup>9</sup> William Stallings, Data and computer communications, Chapter 3, transmisia datelor,

[http://ftp.utcluj.ro/pub/users/cemil/prc/Chapt\\_3ro.ppt#256,1](http://ftp.utcluj.ro/pub/users/cemil/prc/Chapt_3ro.ppt#256,1), William Stallings Data and Computer Communications

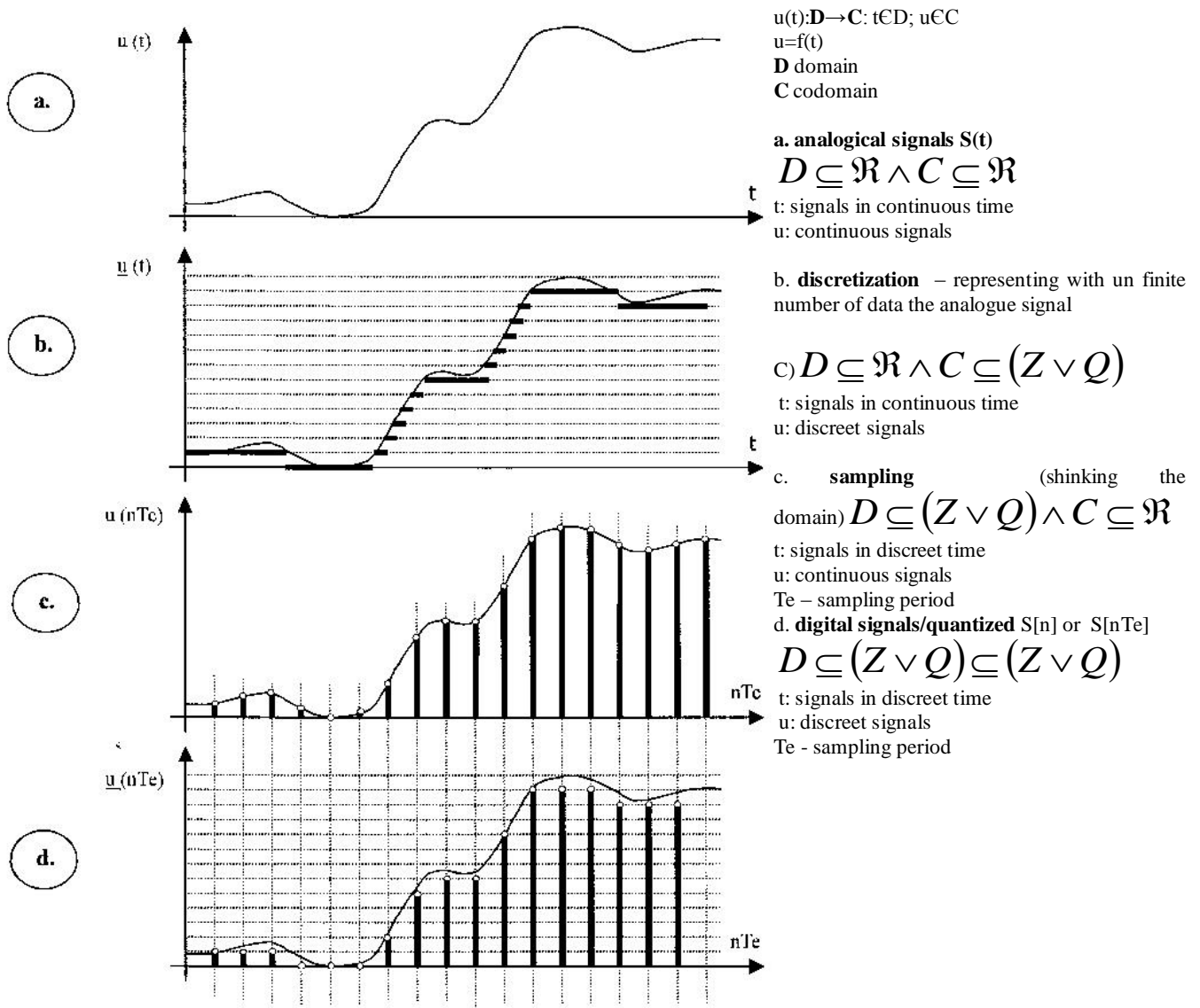


Fig.1<sup>10</sup>. Main types of signals as time function

After an complete cycle,  $t=T$ :

$$\varphi = \omega T = 2\pi$$

$$\omega = \frac{2\pi}{T} \quad (3)$$

<sup>10</sup> Fig.1. este preluată din Ioan P. Mihu, Despre semnale și sisteme din volumul Procesarea numerică a Semnalelor, pg.3

The general form with the initial phase:

$$x(t) = A \sin(\omega t + \phi), \text{ la } t=0, \phi_0=0 \quad (20)$$

*(we consider our signal with initial phase 0)*

$$x_1(t) = A_1 \sin(\omega t + \phi_1)$$

$$x_2(t) = A_2 \sin(\omega t + \phi_2)$$

Then the phase shift of  $x_2$  by  $x_1$  is:

$$\Delta\phi = (\omega t + \phi_2) - (\omega t + \phi_1) = \phi_2 - \phi_1 \quad (4)$$

If two signals have rigorously the same frequency then are coherent signals.

where:

**A** the amplitude of the oscillation (it is an positive number and has the physical dimension of  $x(t)$ ) [um  $x(t)$ ]

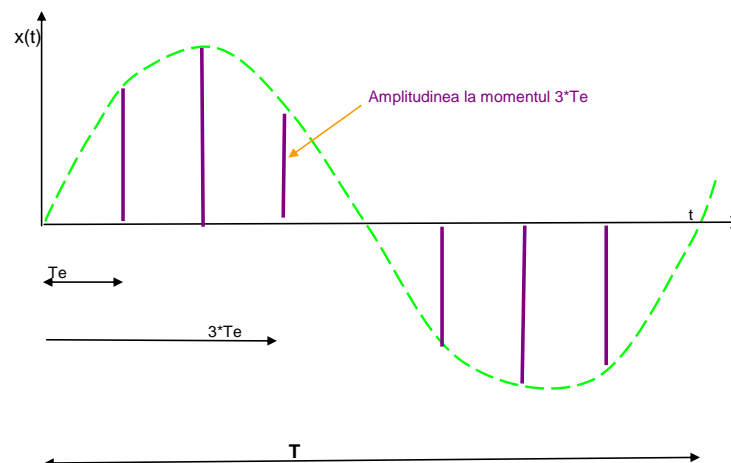
**T period.** Represents the time duration after that the  $x(t)$  gets same values, in the same sense [seconds; hours; months; years, etc]

**f** frequency of the signal  $x(t)$ , the inverse of the T. Show as many times the cycle is repeated in the time unit of T [Hz]

**$\omega$**  pulsation or angular speed [rad/s]

**$\phi$**  observed signal phase  $x(t)$ . it is the argument of the sinus function and has the value of an angle, its position in time is measured in [rad];

The observed data (including labour market data) and provided by statistical sources could be interpreted as the amplitudes of the signal in expression digitalised/quantified, already sampled. (see Fig.2.)



$x(t)$  = variable from the labour market with signal representation

**$T_e$  sampling period** (the minimum period of time for which are provided the data on labour market – in our case monthly frequency)

**T** period – in our case 1 year 1;

**A = A(t)** amplitude is function of the measurement moment, at any  $T_e$

**Fig. 2. Signal in expression digitalised/quantified (including some labour market data) – sinusoidal representation**

We emphasise that the ICA method was developed in technical sciences like electronics, imagistic and telecommunication. It is connected with another signal important property is interference. Interference represents „a physical phenomenon that combines the electromagnetic, sound or optical signals through addition or subtraction according to their phases.”<sup>11</sup> The „superposing of the signals from the same

<sup>11</sup> <http://www.scribube.com/tehnica-mecanica/SEMNALE-ELECTRICE93336.php>

frequency band (difference between maximum and minimum of the frequency's,  $f_{\max} - f_{\min} = \text{LBs}$ )<sup>12</sup>. This mixing phenomenon of the signals could distort the original signal. "Under certain conditions, the signals underlying measured quantities can be recovered by making use of ICA. ICA is a member of a class of *blind source separation* (BSS) methods."<sup>13</sup>

### 3. Method Independent Component Analysis ICA

The modelling and prediction of the dynamics of the macroeconomic factors using the Independent Component Analysis – ICA is still at beginning. Hyvärinen, Karhunen and Oja<sup>14</sup> affiliate this method among the methods „for finding underlying factors or components from multivariate (multidimensional) statistical data”. This method is strongly differentiated from others through the two essential hypotheses that are simultaneously realised: the component looked for are **both independent** (one component doesn't offer any information about the other component) in statistical sense and is **non-Gaussian** (have not a normal distribution). This method allows the linear representation only based on multivariate signals/data offering a „learned” representation without supervisor, as neuronal calculus type. The intrinsic data message exploration includes the ICA method under the **data mining** and **exploratory data analysis** methods. First mention of this method was signalled in **1980**, by J. Herault, C. Jutten, B. Ans<sup>15</sup>, in neuronal signal processing data. The next step in this method development was represented signal separation of sources solutions (including BSS blind sources separation) by J.F. Cardoso, P. Comon<sup>16</sup>, J.L.Lacoume, A. Cichocki and R. Unbehauen. The exponential growth of the computers power brings in attention of the users from different fields those techniques developed in more „strict technical fields” [brains imagistic, imagines processing, signal deconvolution, telecommunications – lately other signals from mobile communication CDMA (Code-Division Multiple Access), etc.]. The **Fast Independent Component Analysis (FICA) or FastICA** algorithm (created by Hyvärinen, Karhunen, Oja) enhanced the popularity of the method ICA because of its efficiency and new applications like: Hopfield networks, self organised maps Kohonen SOM, JADE algorithm, Cichocki-Unbehauen algorithm, etc. **FastICA** could be applied now on different software environments (MATLAB 5 /5.2., C++/C, STATISTICA 8.0, as well as dedicated soft). The STATISTICA 8.0. environment offer two options: the simultaneous extraction variant (parallel) and the successive extraction variant (one unit at the time- artificial neuron with the weight of an vector, neuron under the property of being able to learn – named also deflation). We highlight the expanding tendency of ICA's borrowing in economics<sup>17</sup>. The financial domain registered the first applications of those techniques especially on parallel data similar with the econometric applications in parallel time series separations, decomposing in independent component which offers a new perspective over the **data structure**. In view to apply those techniques are respected the independence hypothesis and non-Gaussianity of the components/factors that describes in a linear representations the measured multivariate data.

<sup>12</sup>Jacob, Sisteme și tehnici multimedia, [http://andrei.clubcisco.ro/cursuri/5master/tagcmrv-stm/01\\_stmm1\\_intro.ppt#273,33,Slide 33](http://andrei.clubcisco.ro/cursuri/5master/tagcmrv-stm/01_stmm1_intro.ppt#273,33,Slide 33)

<sup>13</sup>Stone, J.V. (2005): *A Brief Introduction to Independent Component Analysis* in Encyclopedia of Statistics in Behavioral Science, Volume 2, pp. 907–912, Editors Brian S. Everitt & David C. Howell, John Wiley & Sons, Ltd, Chichester, 2005 [ISBN 978-0-470-86080-9](https://doi.org/10.1002/9780470860809.ch47)

<sup>14</sup>Hyvärinen, Karhunen, Oja, Independent Component Analysis, 2001 John Wiley & Sons, pg.1.

<sup>15</sup>Cited from „Martin Sewell, Independent Component Analysis, Department of Computer Science University College London <http://www.stats.org.uk/ica/>: „ANS, B., J. H'ERAULT, and C. JUTTEN, 1985. Adaptive neural architectures: Detection of primitives. In: Proceedings of COGNITIVA '85. pp. 593–597. // H'ERAULT, J., and B. ANS, 1984. Circuits neuronaux à synapses modifiables: D'ecodage de messages composites par apprentissage non supervis'e. Comptes Rendus de l'Acad'emie des Sciences, 299(III-13), 525–528. // H'ERAULT, J., C. JUTTEN, and B. ANS, 1985. D'etecion de Grands Primitives dans un Message Composite par une Architecture de Calcul Neuromim'etique en Apprentissage non Supervis'e. In: Actes du X'eme colloque GRETSI. pp. 1017–1022.”

<sup>16</sup>COMON, Pierre, 1994. Independent Component Analysis, A New Concept?, Signal Processing, 36(3), 287–314.

<sup>17</sup>K. Kiviluoto și E. Oja, Independent component analysis for parallel financial time series. În *Proc. ICONIP'98, vol.2, pg. 895-898, Tokio, Japan, 1998.*

**Independent Component Analysis ICA/ Nongaussian factor analysis**

“A generative model – it describes how the observed data are generated by a process of mixing the components

*Sj*”[book, ICA, Hyvarinen, Karhunen, Oja, pg.151]

**Assumption**

the components  $s_i$  are statistically *independent*-

- the key to estimating the ICA model is nongaussianity
- a gaussian variable has the largest entropy among all random variables of equal variance. **Negentropy** is in some sense the optimal estimator of nongaussianity

the independent component must have *nongaussian* distributions

we observe  $n$  linear mixtures  $x_1, \dots, x_n$  of  $n$  independent components

$$x_j = a_{j1}s_1 + a_{j2}s_2 + \dots + a_{jn}s_n, \text{ for all } j.$$

$x_j(t)$ , observed values, mixture  $x_j$ , random vector

$s_k(t)$  independent component (“**source**” means here an original signal, *latent variables*, meaning that they cannot be directly observed.), is a random variable, instead of a proper time signal, **nongaussian**

**The ICA model:**

$$\mathbf{x} = \mathbf{A}\mathbf{s},$$

$\mathbf{A}$  mixing matrix

$$\mathbf{x} = \sum_{i=1}^n \mathbf{a}_i s_i.$$

$$\mathbf{s} = \mathbf{W}\mathbf{x} = \mathbf{A}^{-1}\mathbf{x}$$

$\mathbf{W}$  un-mixing/ separation matrix

ICA consists in estimating both the  $s(t)$  and  $\mathbf{A}(t)$ , ( $j=k$ )

**Ambiguities of ICA**

1. We cannot determine the variances (energies) of the independent components
2. We cannot determine the order of the independent components.

**Preprocessing for ICA**

**Centering** - the observed signals are centred around their means **so that the transformed signals have zero mean**

**Whitening** - linearly transforming the observed signals  $X$  into a new set of variables, which **are uncorrelated and their variances equal unity**

**The FastICA Algorithm:** algorithm for maximizing the contrast function - nonquadratic function  $G$

**Deflation** - which extracts one principal component at a time.

**Parallel** or Multiple Extraction and its multi unit version.

<sup>18</sup> Hyvärinen Aapo and Erkki Oja, Independent Component Analysis: Algorithms and Applications Neural Networks Research Centre, Helsinki University of Technology, pg.2

## 4. Data

In view to find underlying factors using the Independent Component Analysis on labour market we make the application on unemployment rate in monthly variation, analyzed in non-seasonal trends. In our case the time series  $x_i(t)$ ,  $i=1$  to 141 months,  $T= 1$  year, in view to isolate the effects of policies across the UE 27 using only one indicator the unemployment rate seasonally adjusted, in monthly variation at EU27 level between January 2000-September 2011. The data source is EUROSTAT, indicator [une\_rt\_m]: „Unemployment rate, monthly average, seasonally adjusted data (see Annex 1, Metadata notes), all persons (15-74 years), in (%), resulting 141 months/cases. (see Fig.1 ). In the last decade there are similitudes in the variation of unemployment rate in stages: convergence, minimum and divergence. It is evident that before February and March 2008 (respectively the month 98 and 99) there were evident some characteristics for entire EU27 :

- a. An “homogene” behaviour:
  - an convergent tendency of the diminishing the level of unemployment rate around the 6.8% UE27 mean (respectively the month 98 and 99) – as the best performance period for the labour markets in terms o unemployment policies results;
  - an convergent tendency to maximise the level o unemployment rate around the 9.7% UE27 mean (January and May 2010 respectively the month 121 and 125) – as the worst performance period for the labour markets in terms o unemployment policies results. This period could represent the maximum crises period manifestation;
- b. An “un-homogene” behaviour: differentiated behaviour for the EU27 states regarding the performance of labour market performance considering that the decreasing tendency of the unemployment level (for at least 3 months) could be an indicator of the recovery initiating. Evidently after January and May 2010 there is visible a split in countries behaviour- some of them are decreasing the unemployment rates but other it still increases.

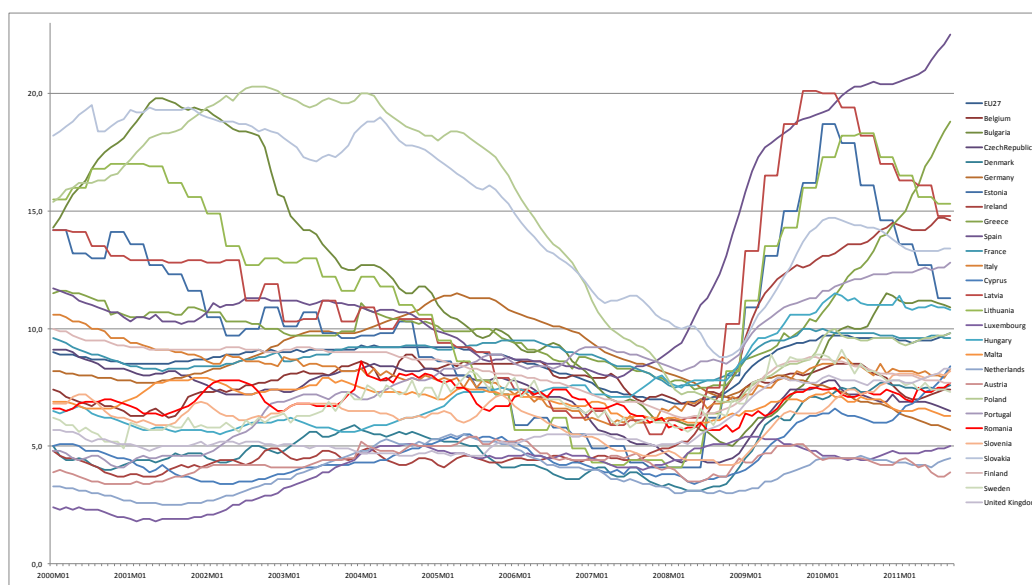


Fig. 3. The original unemployment time series (141 months).

## 5. The Model

In our model unemployment rate is treated as signal of the performance of the labour market functioning performance. Applying the ICA method, we consider that there are multilevel interactions between labour markets and consequently common structures possibly to be identified. Under this perspective there are likely to identify common factors (like crises with sudden effect on labour market functioning, next to seasonal variations due to hollydays and other annual variations). One limit of this model is given by the consideration of “instant” variation and not considering inertia,

delays, etc. Based on the perspective that the interdependency between the European economies is increasing, and then the unemployment rates at national level **are not “pure” but mixed signals** using this method could be an high potential to surprise also the effect of labour market policies of every country (passive and active measures). In this context we try to apply ICA method in view to **isolate the new clusters** generated under the actions of “European” underlying factors, grouped by a common “behaviour”.

### 5a. Pre-whitened data using PCA

The original 27 dimensional signal was projected to the subspace spanned by 11 principal components (covering 99,0697% from total variance of Eigenvalues, see Annex 2\_Table 1a and b, Fig.1). The level of the Eigenvalue indicate also the importance (rank) of the principal component, PCA 1 captures 37,918 % , the PC 2 captures 31.808%, PC 3 captures 19.339%, etc. of the variability in the data. Choosing the number of principal components could be a problem. If we apply a threshold of minim 90% variance of the Eigenvalues then the minimum PCA components will be 4 that cover 92.8272% from total variance of it. So, from the original space of 27 dimension we reduce to an interval of 4 to 11 variables with less redundancy, that conserves as good as possible the initial characteristics.” In PCA the **redundancy is measured by correlations** between data elements, while in ICA the much richer concept of independence is used, and in ICA the reduction of the number of variables is given less emphasis. Using only the correlation as in PCA has the advantage that the analysis can be based on second-order statistics only, In connection with ICA, PCA is a useful pre-processing step...(for ICA first step) data can be pre-processed by whitening, removing the effect of first and second order statistics”<sup>19</sup>.

*Using the STATISTICA 8.0 software we apply PCA as multivariate exploratory technique under variants offered by „STATISTICA PLS”<sup>20</sup> with the state-of-the art NIPALS algorithm for building PLS models (Geladi and Kowalski, 1986). The algorithm assumes that the X and Y data have been transformed to have means of zero. This procedure starts with an initial guess value for the t-scores u and iteratively calculating the model properties in separate and subsequent steps”. Applying NIPALS algorithm with Maximum number of iteration: 50, Convergence criterion=0.0001, Fitting method =Number of components by cross validation V-fold (V value 7), resulted 11 components, noted PCA 1 to PCA 11. The variable importance as a measure of the maximum power of component representation is offered by the model with 11 PCA (see annex 2\_Table 2).*

In Annex 2\_ **Fig. 2 a** and **b** there are presented the case wise data diagnostics in view to **detect the outliers** through: “Hotelling T<sup>2</sup> Control Chart” and “Distance to Model Chart”. The outlier’s analysis reflects that for the last decade there become visible an abnormal tendency in unemployment, localised in the last intervals 140-141 months, corresponding for the months August-September 2011, the end of our series.

As a measure of increasing interdependency between the European economies is the growing tendency of **clustering among variables** unemployment rates for the EU27. In Annex 2\_Table 3 is presented with red the significant correlation coefficients between the selected variables. There are relations (positive or negative) one to many, for each country. Austria, Belgium, Netherland and Poland indicate greater independence (over 8/26 insignificant coefficients). On the other extreme Czech Republic, Hungary, Denmark, Italy, Lithuania, Malta, Slovenia indicate labour market sensitiveness (only 3/26 insignificant coefficients). As particular case **Romania indicates positive dependency with all countries UE26**. As a detail, Romania’s highest dependency of the unemployment variation is with France - explains 35,5% from original variability and indicates through the regression coefficient 0.599% as monthly medium increase, as a measure of the "proportional" to each other are two variables.

<sup>19</sup> Hyvärinen A., Karhunen, Oja, Independent Component Analysis, 2001 John Wiley & Sons, Chapter 6. Principal Component analysis and whitening, pg. 125

<sup>20</sup> *Partial Least Squares (PLS)* (also known as Projection to Latent Structure) is a popular method for modelling industrial applications. It was developed by Wold in the 1960s as an economic technique, but soon its usefulness was recognized by many areas of science and applications including *Multivariate Statistical Process Control (MSPC)* in general and chemical engineering in particular.



**Relations between original variables could be described also in the** new coordinate system resulted through PCA applying. This system offers also an image regarding the way in which the original variables are correlated to each other and their influence in determining a component. In **Annex 2\_Table 5 is presented the Correlations of the variable „country” with each factor PCA.** Also „variables placed close to each other influence the PCA model in similar ways, which also indicates they are correlated”<sup>21</sup>To illustrate better this is presented in **Annex 2\_ Fig.3. the line plot (p1) of the variable “country” against the loadings of PC1.** In reference with PCA1 Malta is the less influential role and UK plays the most influential role in determining the first PC1.

In Fig. 4 we identify some possible clusters, after the projection of every input vector “country” into subspace, with **orthogonal factors/component**, given by the PCA1 (p1) and PCA2 (p2) – components that capture almost 70% variability in the data. It is possible to delimitate the following clusters:

- Positive correlation with both factors PCA1 and PCA2 is registered for:  
*First group: Lithuania, Slovenia, Estonia, Latvia, Greece, France, Denmark, Romania.*
- Positive correlation with PCA1 and negative correlation with PCA2  
*Second group (more with the PCA2, uncorrelated with PCA1): Finland, Czech Republic, Italy*
- Positive correlation with PCA2 and negative (Germany at this extreme) correlation with PCA1 (Malta the other extreme): *Poland, Bulgaria, Slovakia.*

In **Fig.5 a and b** there is presented the matrix plot of the data variables „countries” projections in 11 dimensional subspace and 4. Regardless the changing of the number of dimensions the loadings are the same.

#### **5b. Results: Finding underlying factors using the Independent Component Analysis on labour market – application on Unemployment rate in monthly variation**

Applying PCA we obtained uncorrelated components noted as PCA 11 (Annex 3 fig.1a.), but not independent. In Annex 3\_Table 6a and b is presented that the PCA 11 (and also the variant PCA 4) are uncorrelated with their covariance is 0.” The main reason why we calculate the ICA’s for two dimensions 11 and 4 is sustained but the comment *“in real world application is that there is no prior knowledge on the number of independent components. Sometimes the Eigen values spectrum of the data covariance matrix can be used, but in this case the Eigen values decreased rather smoothly without indicating any clear signal subspace dimension. Then the only way is to try different dimensions. If the independent components that are found using different dimensions for the whitened data are the same or very similar, we can trust that they are not just artefacts produced by compression, but truly indicate some underlying factors in the data.”*<sup>22</sup>

Another property is that if they are independent then they are uncorrelated but uncorrelatedness doesn’t imply independence. In the literature there are some considerations useful like:

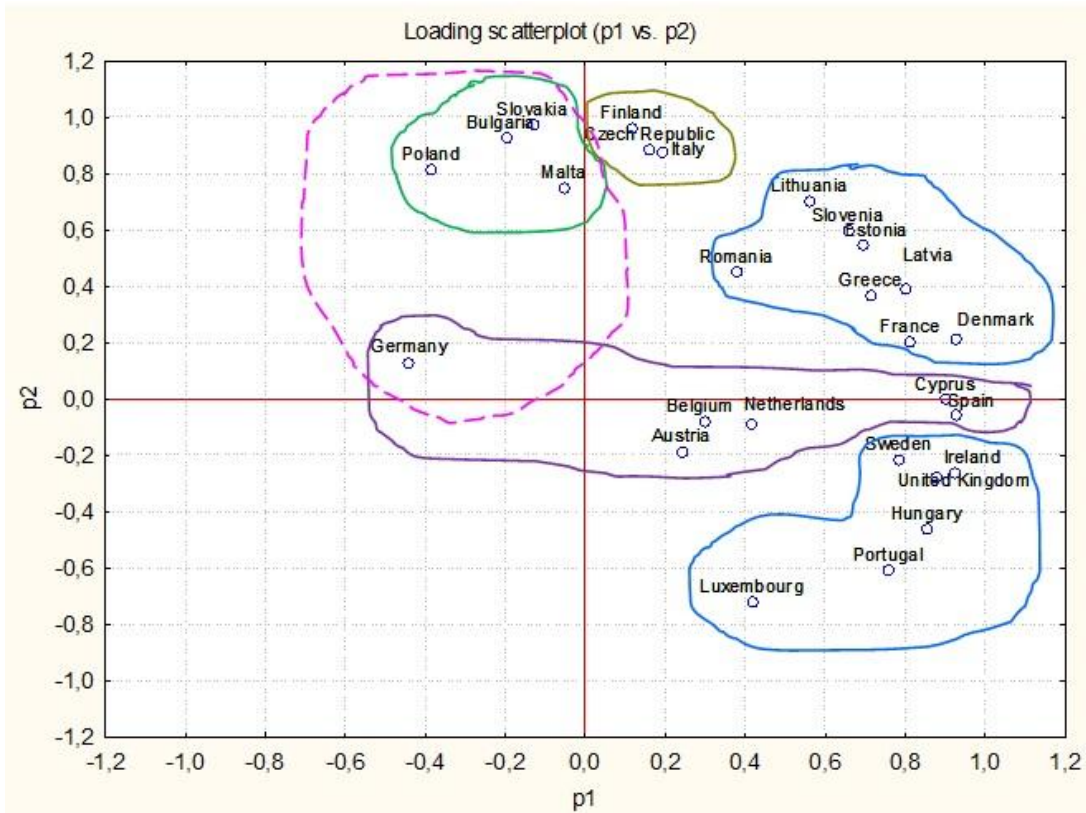
- Whiteness, a slightly stronger property than uncorelatedness of zero mean **vector means that its components** are uncorrelated and their variance equal unity.- the covariance (as well as correlation matrix) matrix equals the identity matrix;<sup>23</sup>
- ICA: **factors are called “sources” and learning is “unmixing”.** Since latent variables assumed to be independent, trying to find linear transformation of data that **recovers independent causes**<sup>24</sup>.

<sup>21</sup> [STATISTICA 8.0. Electronic Manual]

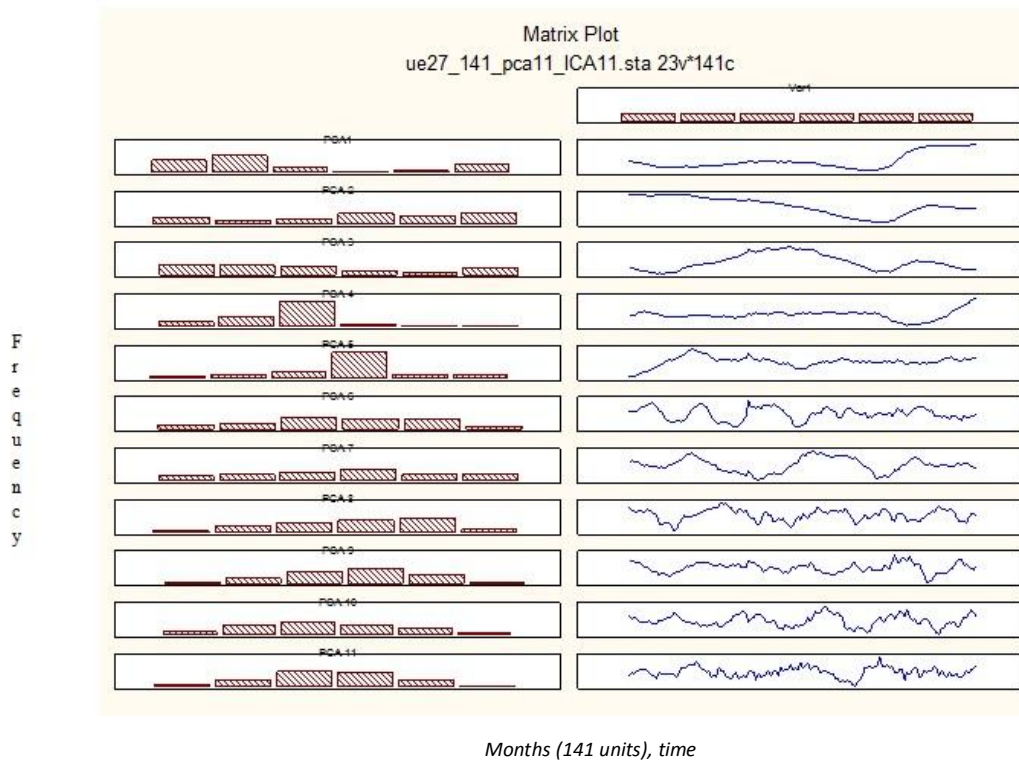
<sup>22</sup> Hyvärinen A., Karhunen, Oja, Independent Component Analysis, 2001 John Wiley & Sons, pg.442

<sup>23</sup> Hyvärinen A., Karhunen, Oja, Independent Component Analysis, 2001 John Wiley & Sons, pg.159

<sup>24</sup> CSC2515: Lecture 8 Continuous Latent Variables, Lecture 9: Continuous, Latent Variable Models  
<http://www.cs.toronto.edu/~hinton/csc2515/notes/lec7middle.pdf>



**Fig.4. Variable „country” projected in the subspace of the PCA1 and PCA2**



**Fig.5. a. Data variables „countries” projections in 11 dimensional subspace**

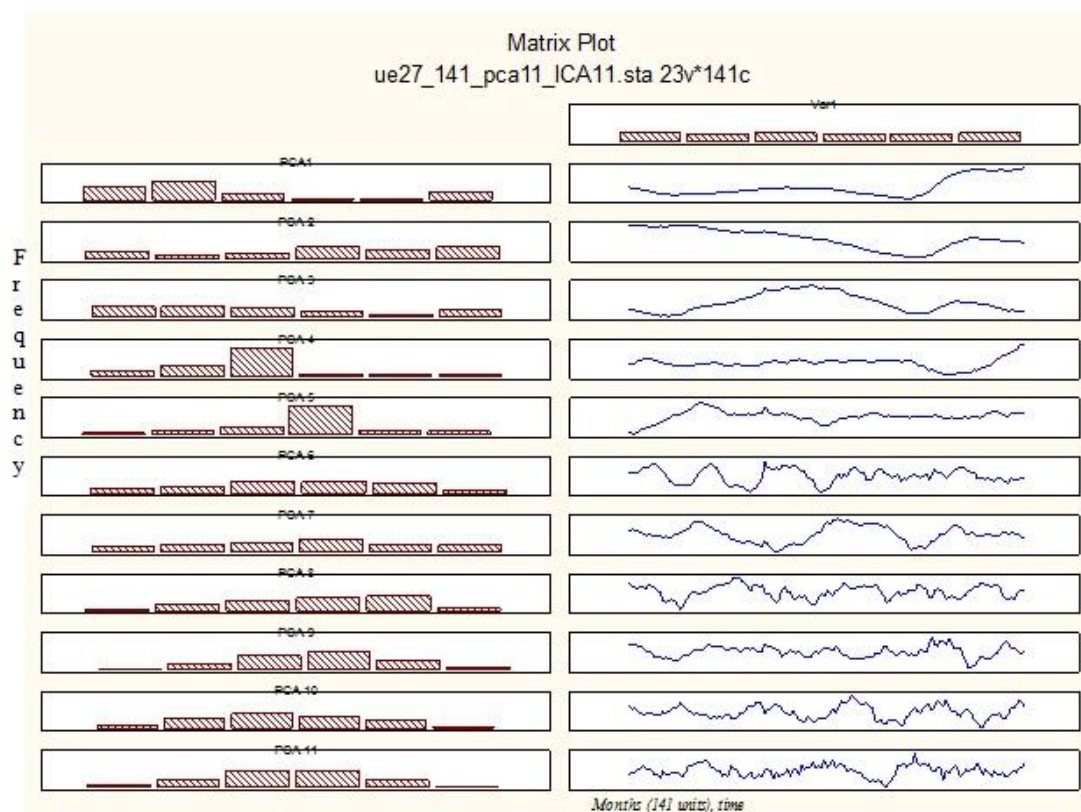


Fig.5. b. Data variables „countries” projections in 4 dimensional subspace

- note that the **components need not be orthogonal, but that the reconstruction is still linear**. ICA is typically posed as an optimization problem. ICA is based on nongaussianity maximization.”<sup>25</sup>

We apply FastCIA algorithm, separate the sources that are maximally independent in view to find non-Gaussian projections as dimensions which directions are meaningful for the decision.

The PCA11, PCA4 components are treated as mixed factors, that acts simultaneously, under the assumption of the statistical property of signals that the sources signals (noted in our case as ICA for 11 components and ICA for 4 components) **are at each time, statistically independent** (independent in the sense that information about the value of one component it not helps in determines the value of other). As it is mentioned in theory “We assume that each mixture  $x_i$  as well as each independent component  $s_j$  is a **random variable (have 0 mean, variance 1, ,** instead of proper time or time series,,,,,We also neglect any time delays that may occur in the mixing, which is why this basic model is often called the instantaneous mixing model”<sup>26</sup>. The obtained component ICA’s are 0 mean and 1 variance”<sup>27</sup> and describes the independent causes. In reality, however, the mixing coefficients (Annex 3 Table 3b) are unknown. Therefore it is the task of *ICA* to find estimates of the mixing coefficients (see Annex 3 Table and b) before we can extract the signals  $s_1$  and  $s_2$ .

There were estimated 11 ICA for the first variant and respectively 4 ICA for the second variant, using the FastICA Algorithm from STATISTICA 8.0. Fast Independent Component Analysis (FICA), with the

<sup>25</sup> David Gleich, Principal Component Analysis and Independent Component Analysis in Neural Networks, CS 152 – Neural Networks, 6 November 2003

<sup>26</sup> Hyvärinen A., Karhunen, Oja, Independent Component Analysis, 2001 John Wiley & Sons, pg.151

<sup>27</sup> Hyvärinen A., Karhunen, Oja, Independent Component Analysis, 2001 John Wiley & Sons, pg.159

parallel extraction method (The components are extracted simultaneously), Function used in the approximation to neg-entropy Nonquadratic function G: Log-cosh, normalised variables.

So the result obtained is calculated for the 2 dimensions 11 and 4, of this endeavour is presented in the Fig. 6 a and b, The reconstructed „sources”- ICA11, as independent components, for the reduced space with PCA at 11 (4) dimensions.

In view to shape better the data model, we keep in mind that it describes a structure “specifies a dedicated *grammar* for a dedicated artificial language for (*a specific*) domain, represents classes of entities (kinds of things) ....., (contains) information and the attributes of that information, and relationships among those entities and (often implicit) relationships among those attributes”<sup>28</sup>.

In our analyze trends excludes the seasonal influences and make possible to identify Factors with sudden effect on labour market functioning in general, and over the unemployment variation in particular. Our sources could have different interpretations, like:

**ICA11\_1:** starting point (minimum) august 2005, maximum point august 2006. Visible effects of the Europe transition from EU15 to EU25, following May 2004, 10 more countries joined the EU: the Czech Republic, Estonia, Cyprus, Latvia, Lithuania, Hungary, Malta, Poland, Slovenia and Slovakia<sup>29</sup> are New Members of UE, (Hurricane Katrina struck Florida and the Gulf Coast after the August), increasing the unemployment rate in EMU countries, etc.;

**ICA11\_2:** starting point (maximum) January 2000 – initiating the Lisbon Strategy;

**ICA11\_3:** starting point (minimum) February 2010 – evident effects of the global crises on increasing the unemployment;

**ICA11\_4:** minimum July 2004, maximum October, 2005; ... etc

.....

At this point there is a lot to work and also interpretations are also possible. Maybe our model is not well stabilised because at the dimension variations, the components found are not virtually the same.

Using the found mixing coefficients it is also possible to analyse the original time series and cluster them in groups.

Based on the perspective that the interdependency between the European economies is increasing, and then the unemployment rates at national level **are not “pure” but mixed signals** using this method could be a high potential to surprise also the effect of labour market policies of every country (passive and active measures).

## 6. Final remarks

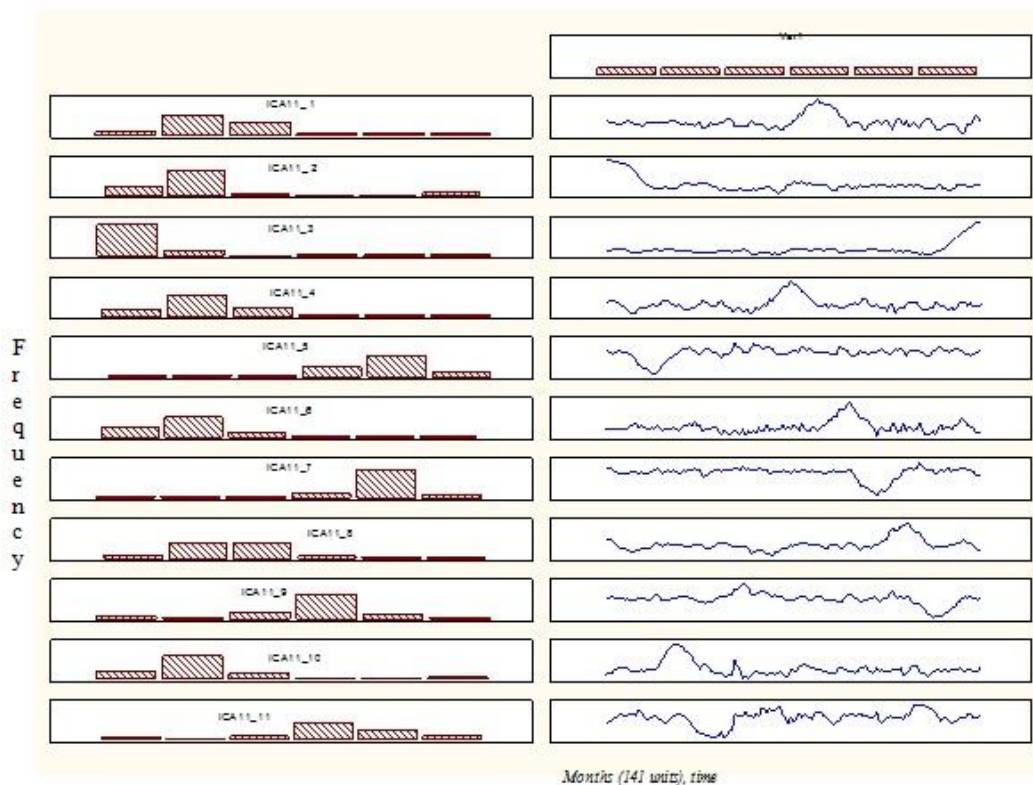
„The ICA success is dependent by the key hypothesis that regards the physical world philosophy approach. This hypothesis sustain that variables or the independent signals are generates by different physical processes.”<sup>30</sup> Stone (2005), with the convention that the term of signal is interchangeable with the term of variable, emphasis independence in the ICA context using the argument that two independent signals do not allow the prediction of one using the other. The empirical experience in the science field (physics, biology, IT, price stocks, etc.) has demonstrated that, without any doubt, in reality most of the measured signals have to be independent signals mixtures. Regardless its limits, “ICA is a very general technique in which are observed random data linear transformed in independent component, maximum independent one regard the other and in the same time they have “ interesting distributions”, this method allow the optimised estimation of latent variables model”<sup>31</sup>. The increasing computing capacity brings in the users attention also these techniques imported from strict technical domains. Beyond the analogy of signal theory with the labour market field, it is still a challenge to interpret the results obtained through this method. The idea to better exploit this relatively new technique represents only the frame and impetus to keep learning.

<sup>28</sup> [http://en.wikipedia.org/wiki/Data\\_model#Data\\_structure](http://en.wikipedia.org/wiki/Data_model#Data_structure)

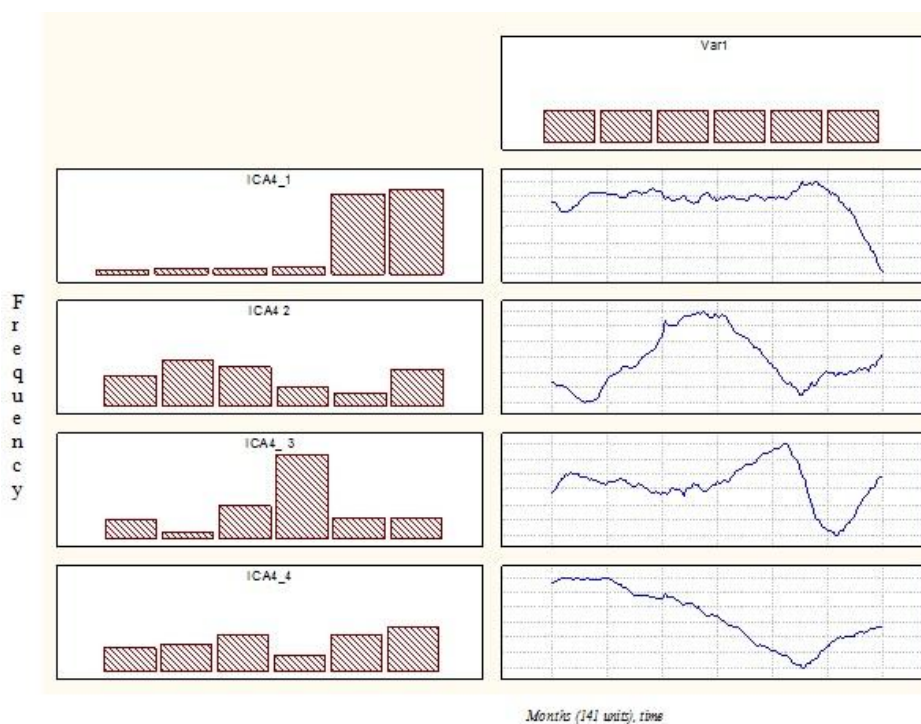
<sup>29</sup> [http://epp.eurostat.ec.europa.eu/statistics\\_explained/index.php/Glossary:European\\_Union\\_\(EU\)](http://epp.eurostat.ec.europa.eu/statistics_explained/index.php/Glossary:European_Union_(EU))

<sup>30</sup> Bell, A.J. & Sejnowski, T.J. (1995). An information maximization approach to blind separation and blind deconvolution, *Neural Computation* 7, 1129–1159.

<sup>31</sup> <http://research.ics.tkk.fi/ica/>



**Fig.6. a. The reconstructed „sources”-ICA11, as independent components, for the reduced space with PCA at 11 dimensions**



**Fig.6. b. The reconstructed „sources”-ICA 4, as independent components, for the reduced space with PCA at 4 dimensions**

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 Eurostat

## Annex 1

### Metadata notes: Unemployment - LFS adjusted series<sup>32</sup>

#### Unemployment - LFS adjusted series

Compiling agency: Eurostat, the statistical office of the European Union, extraction from Metadata.doc

*Unemployed persons* are all persons 15 to 74 years of age (16 to 74 years in ES, SE (1995-2000), UK, IS and NO) who were not employed during the reference week, had actively sought work during the past four weeks and were ready to begin working immediately or within two weeks. Figures show the number of persons unemployed in thousands.

The duration of unemployment is defined as the duration of a search for a job or as the length of the period since the last job was held (if this period is shorter than the duration of search for a job).

Monthly data on seasonally adjusted unemployment rates are published approximately 31 days after the end of the month (average timeliness of 2009 releases).

In the monthly application, the idea is to keep the time series as comparable in time as possible. It means that possible breaks in the LFS series due to changes in the definitions or in the filtering of the micro data have been adjusted: in 1991/1992 there was general definition precision; the gradual implementation of the 'new' unemployment definition following the Regulation (EC) 1897/2000 still leads to backwards revisions while also a general improvement in the micro data filtering of the LFS data from 2001 onwards caused breaks and backwards adjustments. While the original LFS data consists of the raw series as they are recorded at each point of time, the same series are adjusted when they are used as benchmarks for the monthly harmonized time series;

Seasonal adjustment is done by Eurostat for most Member States on a disaggregated level (country by gender by agegroup, indirect approach) using TRAMO/SEATS.

<sup>32</sup> [http://epp.eurostat.ec.europa.eu/cache/ITY\\_SDDS/en/une\\_esms.htm](http://epp.eurostat.ec.europa.eu/cache/ITY_SDDS/en/une_esms.htm)



**Annex 2\_ Table 1a. Information about the PC- principal component model**

Principal Components Analysis Summary (UE27_141.sta)								
Number of components is 11								
99,0698% of sum of squares has been explained by all the extracted components.								
Component	R2X	R2X(Cumul.)	Eigenvalues	Q2	Limit	Q2(Cumul.)	Significance	Iterations
1	0,379186	0,379186	10,23803	0,176926	0,043915	0,176926	S	13
2	0,318089	0,697275	8,58839	0,447203	0,045379	0,545007	S	9
3	0,193390	0,890665	5,22154	0,598256	0,046957	0,817210	S	4
4	0,037607	0,928272	1,01539	0,266034	0,048662	0,865838	S	9
5	0,020227	0,948499	0,54612	0,132572	0,050512	0,883624	S	14
6	0,014435	0,962934	0,38975	0,121133	0,052525	0,897721	S	6
7	0,009726	0,972660	0,26260	0,158079	0,054726	0,913889	S	14
8	0,007098	0,979758	0,19164	0,120777	0,057143	0,924289	S	7
9	0,004579	0,984337	0,12364	0,081423	0,059809	0,930454	S	15
10	0,003673	0,988010	0,09917	0,112983	0,062765	0,938311	S	17
11	0,002688	0,990698	0,07257	0,050971	0,066063	0,941456	UNKNOWN	13

OBS: we use as variable scale the *Unit standard deviations*

Where PC model diagnostics

*R<sup>2</sup>X* the fraction of the explained variation (the larger the greater is significance of the principal component analysed)

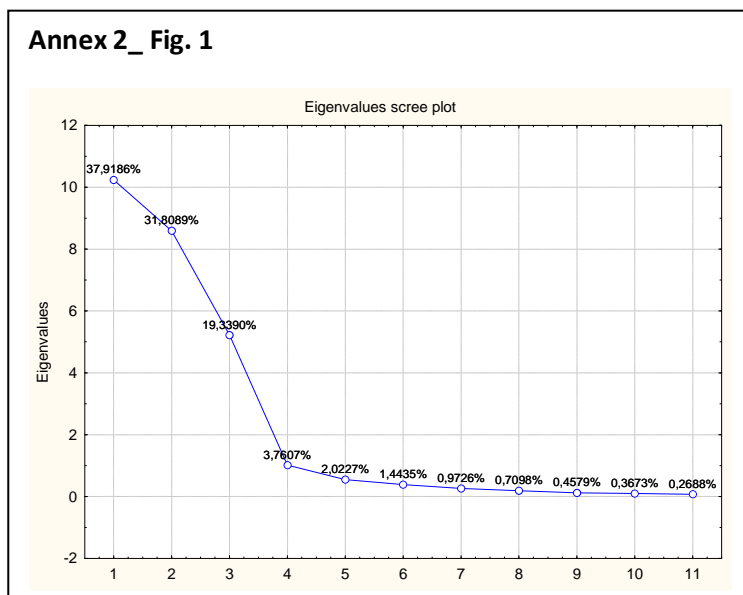
*Q<sup>2</sup>X* the fraction of predicted variation

Because the first principal components respects the rule  $Q^2 > Limit$  then are significant

**Annex 2\_ Table 1b**

Principal Components Analysis Eigenvalues (UE27_141_4PCA.sta)				
Number of components is 12				
Principal Components Analysis sum of variance 27,0000				
	Eigenvalues	% Total variance	Cumulative eigenvalue	Cumulative %
PCA1	10,23803	37,91862	10,23803	37,91862
PCA2	8,58839	31,80885	18,82642	69,72747
PCA3	5,22154	19,33903	24,04796	89,06650
PCA4	1,01539	3,76069	25,06334	<b>92,82720</b>
PCA5	0,54612	2,02268	25,60947	94,84987
PCA6	0,38975	1,44351	25,99922	96,29339
PCA7	0,26260	0,97261	26,26182	97,26600
PCA8	0,19164	0,70979	26,45346	97,97579
PCA9	0,12364	0,45792	26,57710	98,43371
PCA10	0,09917	0,36730	26,67627	98,80101
PCA11	0,07257	0,26877	26,74884	<b>99,06978</b>
PCA12	0,05723	0,21195	26,80607	99,28173

**Annex 2\_ Fig. 1**



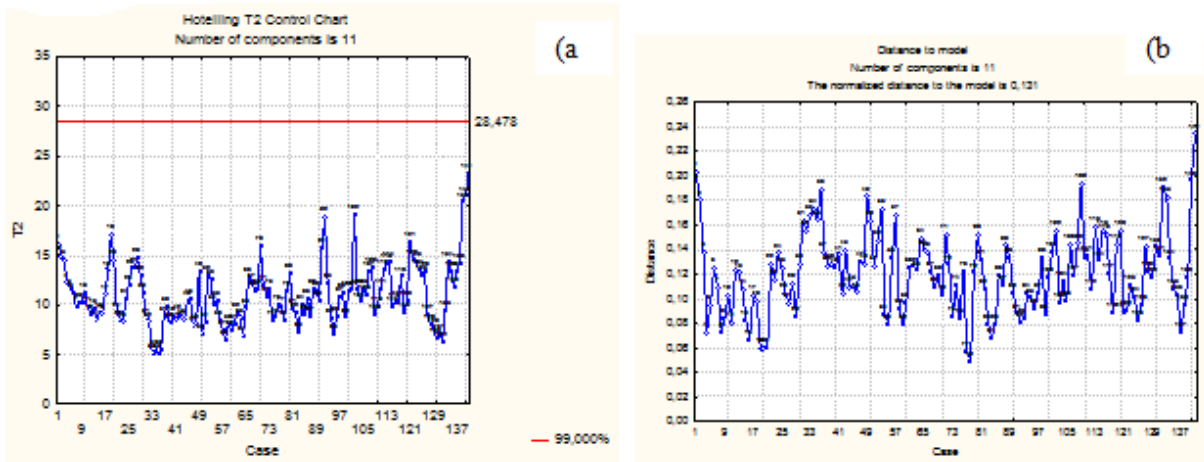
*Note that the first principal eigenvalue captures 37,918 %, the second one captures 31.808% and the third one captures 19.339% of the variability in the data. However, this trend decreases as we add more components to the model.*

Annex 2\_Table 2. Variable importance

Variable	Variable importance (UE27_141.sta) Number of components is 11		
	Variable number	Power	Importance
Romania	25	0,997915	1
Ireland	10	0,997874	2
Austria	22	0,997071	3
Spain	12	0,996412	4
United Kingdom	30	0,994116	5
Luxembourg	18	0,994084	6
Portugal	24	0,993628	7
Estonia	9	0,993354	8
Slovakia	27	0,993246	9
Netherlands	21	0,992238	10
Czech Republic	6	0,992009	11
Finland	28	0,991792	12
Latvia	16	0,991673	13
Lithuania	17	0,991464	14
Hungary	19	0,991440	15
Italy	14	0,991365	16
France	13	0,990970	17
Malta	20	0,990497	18
Belgium	4	0,990273	19
Denmark	7	0,989623	20
Poland	23	0,989229	21
Bulgaria	5	0,988871	22
Greece	11	0,988199	23
Germany	8	0,982553	24
Sweden	29	0,982545	25
Cyprus	15	0,980816	26
Slovenia	26	0,977986	27

Power measures how well a variable is represented by the principal component. All countries EU 27 are well represented by PCA.

Annex 2\_Fig. 2. Case wise data diagnostics in view to detect the outliers



Another data diagnostics using scatterplot of the  $x$ -scores, “The  $x$ -scores are the transformed values of the  $X$  observations in the principal component system. An  $x$ -score with too high a value (i.e., one that deviates substantially from the point of origin) can again be regarded as an outlier or abnormal.” [STATISTICA 8.0. Electronic Manual]



**Annex 2\_Table 3.b. Clustering among the variables (Detail Romania):**

Romania:Belgium:	$y = 3,3722 + 0,4673*x$ ; $r = 0,5155$ ; $p = 0,0000$ ; $r^2 = 0,2657$
Romania:Bulgaria:	$y = 6,5023 + 0,0398*x$ ; $r = 0,2851$ ; $p = 0,0006$ ; $r^2 = 0,0813$
Romania:Czech Republic:	$y = 4,8505 + 0,2955*x$ ; $r = 0,5940$ ; $p = 0,0000$ ; $r^2 = 0,3528$
Romania:Denmark:	$y = 5,7623 + 0,2382*x$ ; $r = 0,5009$ ; $p = 0,0000$ ; $r^2 = 0,2509$
Romania:Germany:	$y = 5,8762 + 0,1257*x$ ; $r = 0,3003$ ; $p = 0,0003$ ; $r^2 = 0,0902$
Romania:Estonia:	$y = 6,4857 + 0,0469*x$ ; $r = 0,2880$ ; $p = 0,0005$ ; $r^2 = 0,0830$
Romania:Ireland:	$y = 6,8493 + 0,0175*x$ ; $r = 0,1070$ ; $p = 0,2067$ ; $r^2 = 0,0114$
Romania:Greece:	$y = 5,689 + 0,1231*x$ ; $r = 0,4319$ ; $p = 0,00000$ ; $r^2 = 0,1865$
Romania:Spain:	$y = 6,6592 + 0,0247*x$ ; $r = 0,1698$ ; $p = 0,0441$ ; $r^2 = 0,0288$
Romania:France:	$y = 1,5919 + 0,5994*x$ ; $r = 0,5941$ ; $p = 0,0000$ ; $r^2 = 0,3529$
Romania:Italy:	$y = 5,8899 + 0,1345*x$ ; $r = 0,2357$ ; $p = 0,0049$ ; $r^2 = 0,0556$
Romania:Cyprus:	$y = 5,9532 + 0,2142*x$ ; $r = 0,3736$ ; $p = 0,00001$ ; $r^2 = 0,1396$
Romania:Latvia:	$y = 6,4577 + 0,0437*x$ ; $r = 0,2878$ ; $p = 0,0005$ ; $r^2 = 0,0828$
Romania:Lithuania:	$y = 6,4786 + 0,0416*x$ ; $r = 0,3099$ ; $p = 0,0002$ ; $r^2 = 0,0961$
Romania:Luxembourg:	$y = 6,7571 + 0,052*x$ ; $r = 0,0942$ ; $p = 0,2663$ ; $r^2 = 0,0089$
Romania:Hungary:	$y = 6,931 + 0,0045*x$ ; $r = 0,0138$ ; $p = 0,8710$ ; $r^2 = 0,0002$
Romania:Malta:	$y = 3,1971 + 0,5388*x$ ; $r = 0,4390$ ; $p = 0,00000$ ; $r^2 = 0,1927$
Romania:Netherlands:	$y = 5,4087 + 0,4012*x$ ; $r = 0,5594$ ; $p = 0,0000$ ; $r^2 = 0,3130$
Romania:Austria:	$y = 4,6589 + 0,5315*x$ ; $r = 0,4582$ ; $p = 0,00000$ ; $r^2 = 0,2099$
Romania:Poland:	$y = 6,1683 + 0,0562*x$ ; $r = 0,4347$ ; $p = 0,00000$ ; $r^2 = 0,1890$
Romania:Portugal:	$y = 6,7807 + 0,0226*x$ ; $r = 0,0908$ ; $p = 0,2840$ ; $r^2 = 0,0083$
Romania:Romania:	$y = 0 + 1*x$ ; $r = 1,0000$ ; $p = ---$ ; $r^2 = 1,0000$
Romania:Slovenia:	$y = 4,7185 + 0,3601*x$ ; $r = 0,5630$ ; $p = 0,0000$ ; $r^2 = 0,3169$
Romania:Slovakia:	$y = 5,5817 + 0,0907*x$ ; $r = 0,4843$ ; $p = 0,00000$ ; $r^2 = 0,2345$
Romania:Finland:	$y = 4,3179 + 0,3186*x$ ; $r = 0,4898$ ; $p = 0,0000$ ; $r^2 = 0,2399$
Romania:Sweden:	$y = 5,3632 + 0,2333*x$ ; $r = 0,3777$ ; $p = 0,00000$ ; $r^2 = 0,1427$
Romania:United Kingdom:	$y = 6,8577 + 0,0187*x$ ; $r = 0,0352$ ; $p = 0,6785$ ; $r^2 = 0,0012$

Where:  $r$  - Pearson correlation coefficient / linear or product-moment correlation

$r^2$  - Adjusted  $r^2$ , coefficient of determination/variance of the model's predictions in total variance

$p$  - result of statistical significance testing

Annex 2\_Table 4.

Variable	Descriptive statistics (UE27_141.sta)				
	Variable number	Valid N.	Mean	Std.Dev	Scaling factor
Belgium	4	141	7,68936	0,681773	0,681773
Bulgaria	5	141	11,64326	4,432225	4,432225
Czech Republic	6	141	7,15532	1,242142	1,242142
Denmark	7	141	5,04965	1,299540	1,299540
Germany	8	141	8,66667	1,476756	1,476756
Estonia	9	141	10,21489	3,791794	3,791794
Ireland	10	141	6,61631	3,772449	3,772449
Greece	11	141	10,36738	2,168294	2,168294
Spain	12	141	12,39504	4,249963	4,249963
France	13	141	8,96525	0,612604	0,612604
Italy	14	141	7,99574	1,083373	1,083373
Cyprus	15	141	4,72482	1,078038	1,078038
Latvia	16	141	11,62128	4,072974	4,072974
Lithuania	17	141	11,70426	4,607011	4,607011
Luxembourg	18	141	4,00284	1,120009	1,120009
Hungary	19	141	7,58014	1,886805	1,886805
Malta	20	141	6,99291	0,503508	0,503508
Netherlands	21	141	3,88014	0,861910	0,861910
Austria	22	141	4,33901	0,532752	0,532752
Poland	23	141	14,17660	4,780027	4,780027
Portugal	24	141	8,17376	2,487157	2,487157
Romania	25	141	6,96525	0,618060	0,618060
Slovenia	26	141	6,23972	0,966354	0,966354
Slovakia	27	141	15,24752	3,298735	3,298735
Finland	28	141	8,30922	0,950105	0,950105
Sweden	29	141	6,86809	1,000880	1,000880
United Kingdom	30	141	5,75461	1,164381	1,164381

Annex 2\_ Table 5.

Loading spreadsheet (UE27\_141.sta) Number of components is 11

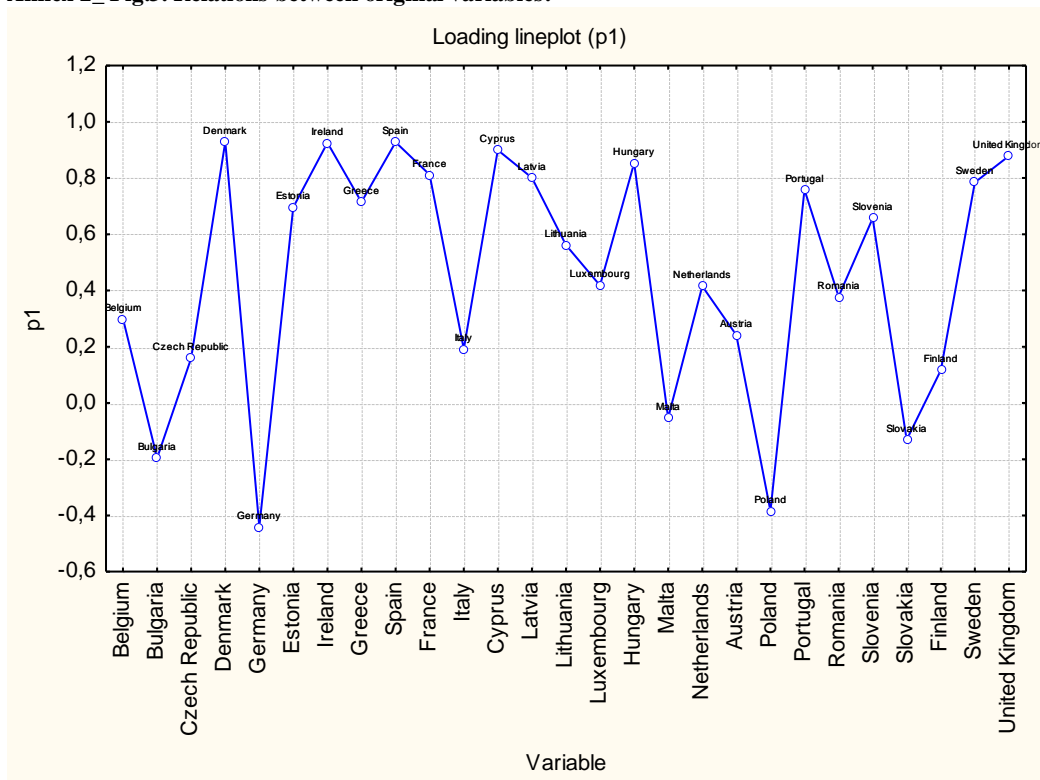
	PCA 1	PCA 2	PCA 3	PCA 4	PCA 5	PCA 6	PCA 7	PCA 8	PCA 9	PCA 10	PCA 11	PCA 12
Belgium	0,3003	-0,0820	0,8733	-0,1689	0,0643	-0,0328	0,0030	0,3000	-0,0415	-0,0595	0,0462	-0,0831
Bulgaria	-0,1959	0,9274	-0,2192	0,1049	0,1582	-0,0322	0,0679	-0,0148	-0,0249	-0,0205	0,0152	0,0418
Czech Republic	0,1591	0,8834	0,3377	0,0052	-0,2351	0,0392	0,0297	-0,0517	-0,0500	0,0893	-0,0439	0,0224
Denmark	0,9289	0,2136	0,0738	0,0200	0,1757	-0,0782	-0,1318	-0,0048	0,0207	-0,0433	-0,1273	-0,0018
Germany	-0,4408	0,1292	0,8695	-0,0392	-0,1171	-0,0312	-0,0186	-0,0066	-0,0133	-0,0141	0,0010	0,0448
Estonia	0,6962	0,5443	-0,2980	-0,3011	-0,0843	0,0634	-0,0561	0,0143	-0,0830	-0,0487	-0,0514	0,0010
Ireland	0,9254	-0,2640	-0,2455	-0,0500	0,1005	-0,0281	-0,0073	0,0023	0,0323	0,0027	0,0151	0,0217
Greece	0,7146	0,3664	-0,1974	0,5297	0,0844	-0,0741	-0,0061	-0,0445	0,0480	0,0332	0,0044	-0,0832
Spain	0,9274	-0,0566	-0,3133	-0,0641	0,1159	-0,0060	-0,0968	0,0233	0,0802	0,0220	0,0308	-0,0066
France	0,8120	0,2026	0,4341	-0,0069	-0,2011	0,0433	0,1682	0,0865	0,0313	0,1226	-0,0805	0,0022
Italy	0,1919	0,8762	-0,2905	-0,0669	-0,2371	0,0316	-0,1144	0,0485	0,0843	-0,0331	0,1132	-0,0334
Cyprus	0,8997	-0,0016	0,0904	0,2882	-0,2363	-0,0313	0,0274	-0,1067	-0,0937	-0,0271	0,0311	-0,1063
Latvia	0,8005	0,3915	-0,3122	-0,2926	0,0082	0,0747	-0,0007	-0,0343	0,0107	-0,0425	0,0093	-0,0194
Lithuania	0,5614	0,7011	-0,3835	-0,1336	0,0339	0,0201	-0,0946	-0,0168	-0,0159	-0,0232	-0,0201	0,0209
Luxembourg	0,4181	-0,7191	0,4634	-0,0101	0,0165	0,0470	-0,2488	-0,0123	0,0855	0,1280	0,0218	0,0198
Hungary	0,8538	-0,4623	-0,1642	-0,0242	-0,0484	-0,0116	0,1017	-0,0049	-0,0859	-0,0686	0,0435	0,0537
Malta	-0,0535	0,7475	0,3936	-0,2873	0,2915	-0,2851	0,0744	-0,0782	-0,0712	0,0965	0,0217	-0,0587
Netherlands	0,4152	-0,0882	0,8512	0,1855	-0,0944	-0,0479	-0,1815	-0,0230	-0,0721	-0,0476	-0,0266	0,0198
Austria	0,2416	-0,1864	0,8766	-0,1690	0,0137	0,0372	0,1559	-0,1682	0,1930	-0,1257	-0,0223	-0,0318
Poland	-0,3842	0,8151	0,3882	0,0735	0,1123	-0,0845	-0,0686	0,0166	0,0488	0,0020	0,0202	0,0115
Portugal	0,7586	-0,6090	0,1436	0,0714	0,0952	-0,0990	0,0092	0,0259	-0,0482	-0,0457	-0,0537	0,0418
Romania	0,3784	0,4517	0,5397	0,2155	0,3094	0,4601	0,0513	-0,0040	-0,0566	0,0162	0,0399	0,0109
Slovenia	0,6568	0,5958	0,1209	0,3231	-0,0329	-0,1906	0,1043	0,1016	0,0724	-0,0278	0,0549	0,1100
Slovakia	-0,1311	0,9732	0,1427	0,0473	0,0166	0,0136	-0,0606	-0,0397	-0,0230	-0,0537	-0,0018	0,0207
Finland	0,1165	0,9618	0,0879	-0,1511	-0,0930	0,0632	-0,0055	0,0432	0,0693	0,0224	-0,0444	0,0017
Sweden	0,7843	-0,2143	0,4683	-0,2506	-0,0274	-0,0434	-0,0162	-0,1433	-0,0509	0,0455	0,1163	0,0521
United Kingdom	0,8789	-0,2782	-0,3308	-0,0982	0,0271	0,0233	0,1356	0,0539	0,0318	0,0703	0,0077	-0,0002

[PCA can also help you to analyze the relationship between the original variables, the way they correlate to each other and their influence in determining the new coordinate system. The quantity at the centre of such analyses is the x-loadings factors. The x-loadings of a principal component with respect to a variable is the cosine of the angle between the directions of that component and the axis of the respective variable. This implies that the more influential a variable in determining a component, the more the variable axis is aligned with that component.

Scatter plots of the loading factors PC 1 and PC 2 ] [STATISTICA 8.0. Electronic Manual]

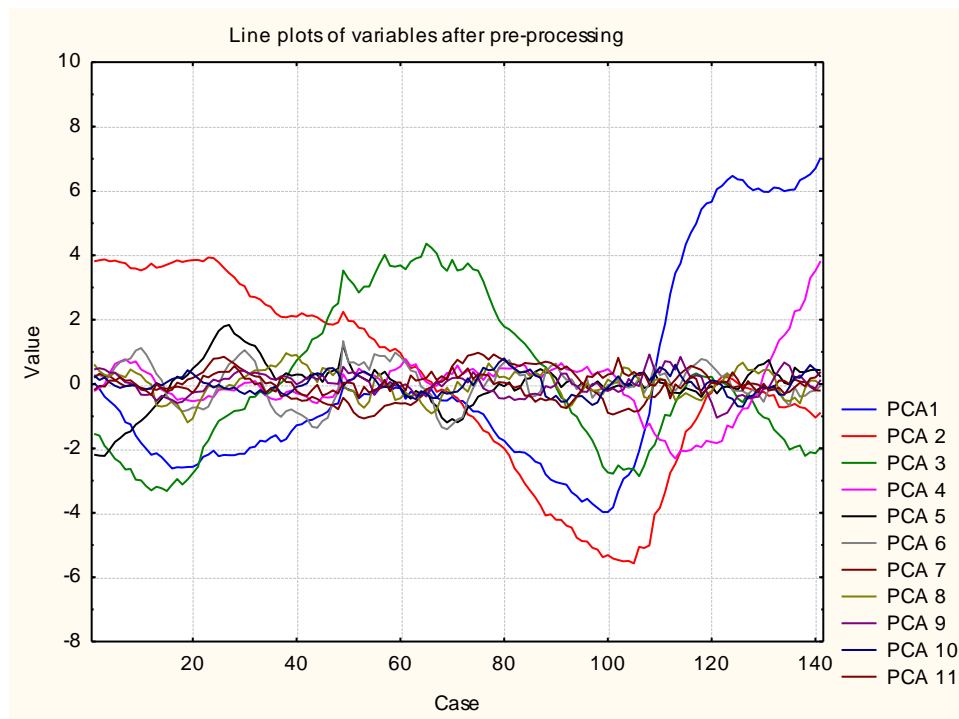
to analyze the relation between the variables and identify the most influential ones in determining the PCA model.

Annex 2\_Fig.3. Relations between original variables.

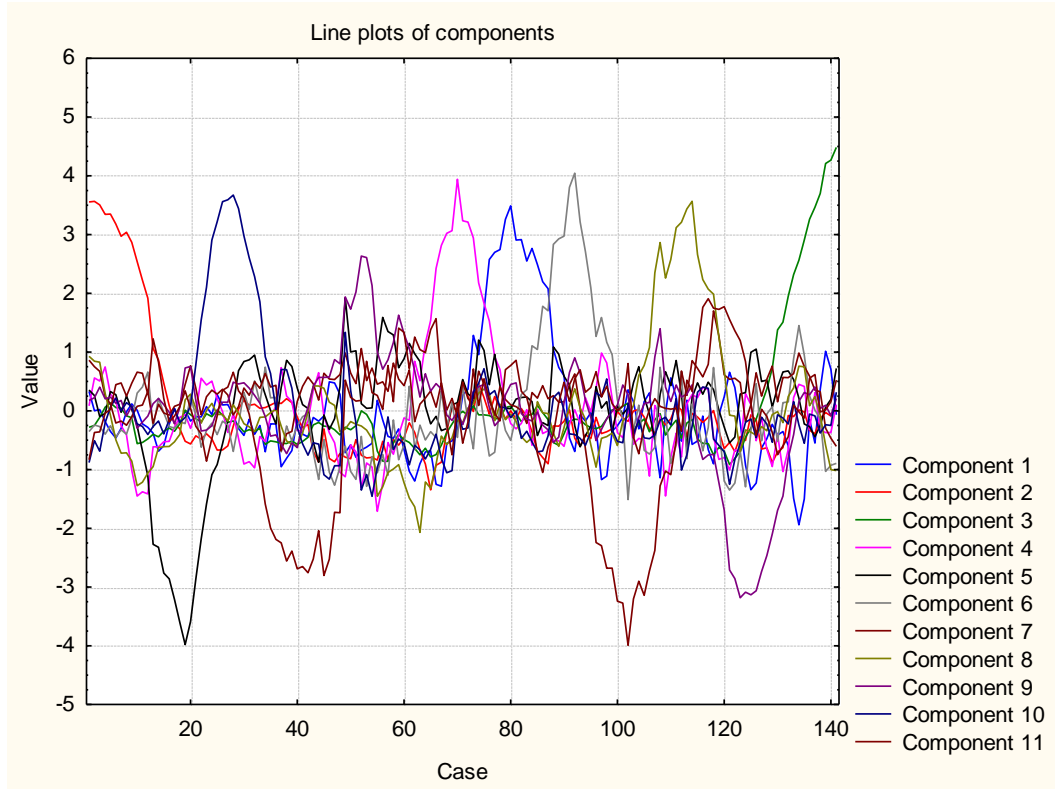


Annex 3\_Fig.1.a

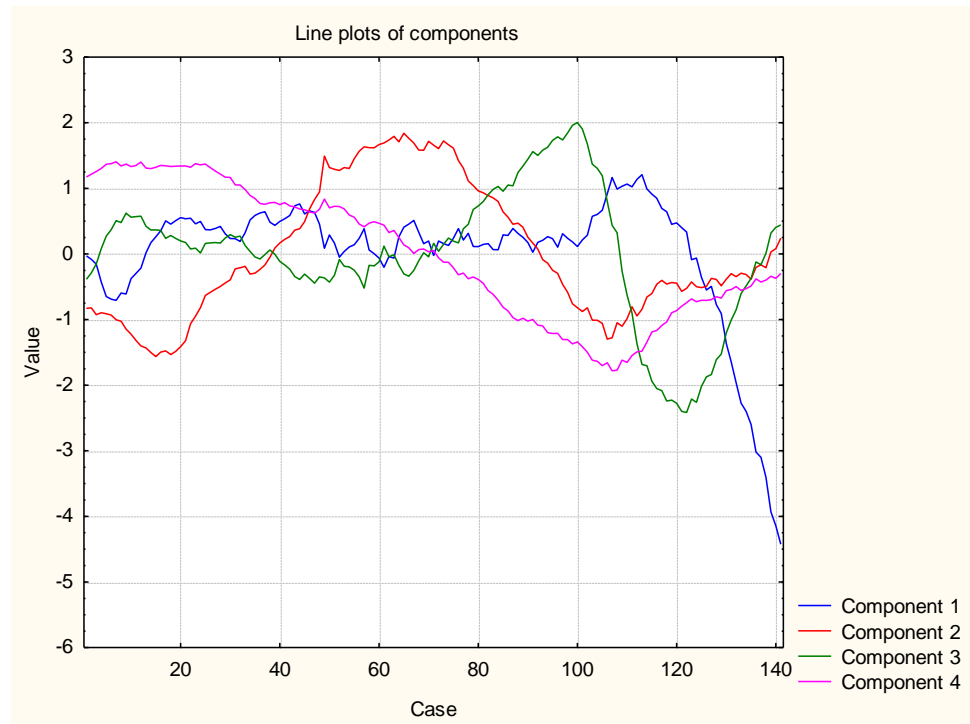
Annex 3



Annex 3\_Fig.1.b



Annex 3\_Fig.1.c





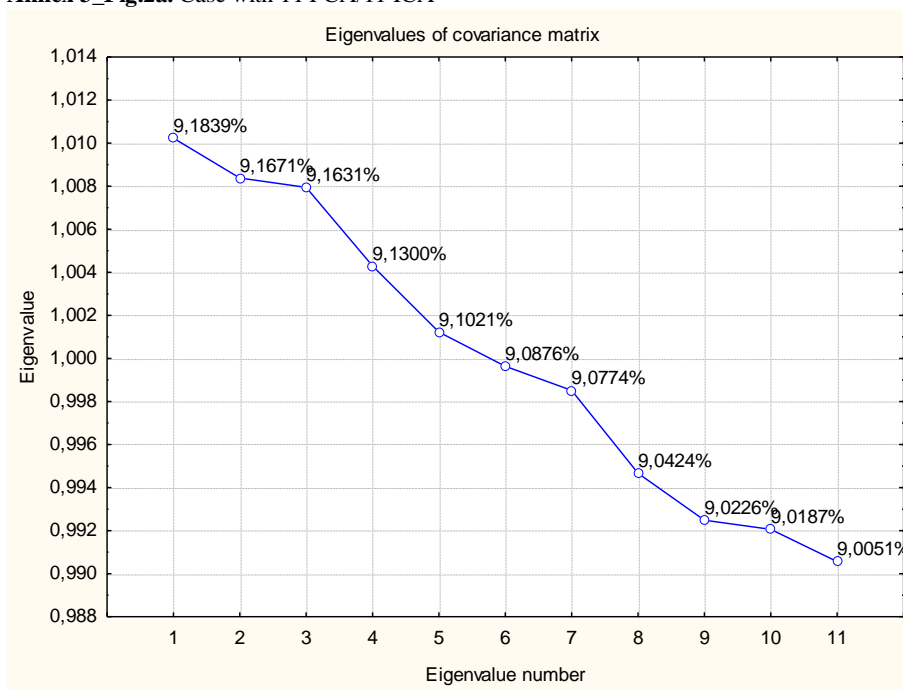
**Annex 3 Table 1**

Eigenvectors of covariance matrix (ue27_14_PCA11_faramediaUe.sta)											
Factor	1	2	3	4	5	6	7	8	9	10	11
PCA1	-0,006570	-0,009134	0,012618	0,074093	-0,197024	0,299039	-0,389047	-0,459584	-0,459139	-0,482496	0,244529
PCA 2	-0,147165	-0,175773	0,126047	0,246640	-0,477937	0,516565	-0,315558	0,053125	0,310157	0,362198	-0,213729
PCA 3	-0,584712	-0,670382	0,367561	-0,000706	0,112692	-0,162353	0,115591	0,003067	-0,083633	-0,101405	0,062386
PCA 4	0,086081	0,061685	0,077737	0,535351	-0,469837	-0,075203	0,468456	0,330700	-0,080564	-0,255767	0,262714
PCA 5	-0,132035	-0,040668	-0,210938	-0,421317	0,018202	0,350328	-0,034631	0,384338	0,307159	-0,185024	0,598626
PCA 6	0,432136	0,033320	0,737030	0,075524	0,195282	-0,057224	-0,194149	-0,051364	0,270284	0,023869	0,328430
PCA 7	-0,221705	0,021122	-0,369226	0,448376	0,094219	-0,353147	-0,217067	-0,337140	0,379404	0,139312	0,394087
PCA 8	0,080711	-0,060597	0,055067	-0,427638	-0,469673	-0,090907	0,414491	-0,540505	0,105304	0,249905	0,193806
PCA 9	0,605857	-0,710828	-0,335969	0,044999	-0,004312	-0,043039	-0,054740	0,033321	0,036108	-0,063135	-0,038658
PCA 10	0,041914	-0,052871	-0,036250	0,091167	0,134110	0,117801	-0,003123	0,134136	-0,580797	0,662070	0,400370
PCA 11	-0,018421	0,029165	0,039387	-0,263845	-0,461921	-0,580526	-0,505404	0,318311	-0,135806	0,036381	0,027205

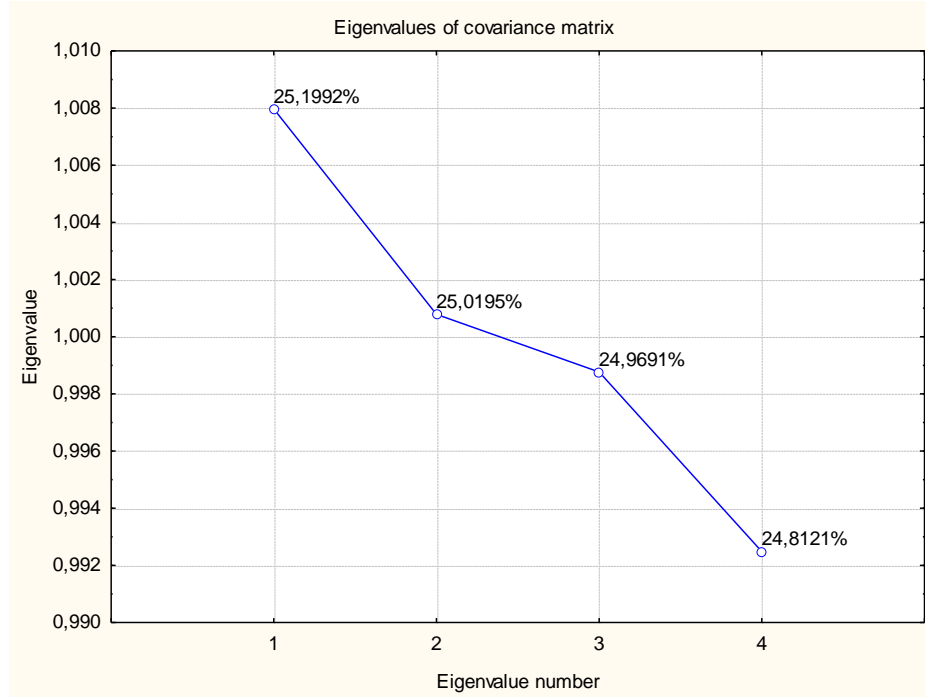
Eigenvectors of covariance matrix (UE27\_141\_4PC)

	1	2	3	4
PCA 1	-0,603891	-0,694122	0,387744	0,056254
PCA 2	0,323247	0,079140	0,532171	0,778487
PCA 3	0,399362	-0,022455	0,672179	-0,623042
PCA 4	-0,609374	0,715142	0,338561	-0,051113

**Annex 3 Fig.2a.** Case with 11 PCA/11 ICA



**Annex 3 Fig.2b.** Case with 4 PCA/4 ICA



**Annex 3 Table 2a**

Non-gaussian matrix (ue27_14_PCA11_farmediaUe.sta)					Estimated Non-gaussian matrix.						
	ICA 1	ICA 2	ICA 3	ICA 4	ICA 5	ICA 6	ICA 7	ICA 8	ICA 9	ICA 10	ICA 11
ICA 1	-0,219246	-0,234663	0,164920	0,112453	0,001232	0,117901	0,490185	0,305702	-0,340904	0,626312	-0,030023
ICA 2	-0,116131	0,340179	-0,399329	0,129538	-0,712979	0,248664	0,129991	0,261979	0,194308	0,001023	0,032976
ICA 3	0,517138	-0,105961	-0,296488	0,712802	0,134443	-0,165877	0,048276	-0,066917	0,164685	0,209096	0,045911
ICA 4	-0,072208	-0,078579	0,397981	0,097637	-0,360900	-0,533854	0,375325	-0,193160	0,145412	-0,176100	0,418455
ICA 5	0,226622	-0,311837	0,469724	0,117107	-0,021170	0,387830	-0,103209	0,520202	0,347229	-0,234706	0,096270
ICA 6	-0,283875	-0,416644	-0,177574	0,177822	0,011851	0,009505	0,362616	-0,105990	0,107959	-0,443366	-0,575748
ICA 7	0,371874	0,564352	0,395987	-0,041263	0,030486	0,065389	0,361939	-0,104612	0,083908	0,022866	-0,479580
ICA 8	0,249231	-0,305383	-0,221357	-0,571103	-0,036629	-0,057413	0,188255	-0,092264	0,551092	0,340510	-0,035040
ICA 9	-0,485290	0,077731	0,258596	0,282148	0,031322	0,193344	-0,255510	-0,383009	0,482997	0,350242	-0,075313
ICA 10	-0,214378	0,295948	-0,204434	0,002787	0,559621	0,187422	0,441332	0,099005	0,246336	-0,186742	0,419789
ICA 11	0,228004	-0,188667	0,020033	-0,011954	-0,162232	0,617387	0,176012	-0,581658	-0,246637	-0,085599	0,258698

**Annex 3 Table 2b**

Non-gaussian matrix (UE27_141_4PCA.sta)				Estimated Non-gaussian matrix.			
	ICA 1	ICA 2	ICA 3	ICA 4			
ICA 1	-0,555627	-0,032717	0,222026	-0,800570			
ICA 2	-0,022943	-0,091327	0,954111	0,284264			
ICA 3	-0,783783	-0,307978	-0,197802	0,501705			
ICA 4	-0,276471	0,946435	0,035376	0,163014			

**Annex 3 Table 3a**

Mixing matrix (ue27_14_PCA11_farmediaUe.sta) Estimated mixing matrix computed from centered data.											
	ICA 1	ICA 2	ICA 3	ICA 4	ICA 5	ICA 6	ICA 7	ICA 8	ICA 9	ICA 10	ICA 11
PCA 1	-0,219139	-0,115109	0,515500	-0,072156	0,225335	-0,283754	0,371623	0,248324	-0,484020	-0,213301	0,227244
PCA 2	-0,234476	0,339123	-0,104645	-0,078003	-0,310638	-0,416919	0,565061	-0,305779	0,077697	0,295185	-0,188123
PCA 3	0,165370	-0,400656	-0,298438	0,399589	0,470836	-0,178865	0,398323	-0,222576	0,259481	-0,204804	0,019685
PCA 4	0,112333	0,131567	0,712269	0,097832	0,116499	0,177810	-0,040679	-0,571122	0,282013	0,002428	-0,011823
PCA 5	0,000274	-0,712336	0,133025	-0,359637	-0,021917	0,011087	0,030018	-0,035857	0,030470	0,557808	-0,163664
PCA 6	0,116686	0,250563	-0,166332	-0,535252	0,388954	0,008743	0,064433	-0,057862	0,194575	0,185630	0,618810
PCA 7	0,489203	0,129535	0,048907	0,377294	-0,104911	0,362657	0,361776	0,188165	-0,255231	0,439802	0,175542
PCA 8	0,304055	0,261296	-0,066882	-0,193017	0,520331	-0,106601	-0,105725	-0,092534	-0,382342	0,098962	-0,581503
PCA 9	-0,341915	0,195470	0,165558	0,145448	0,348837	0,108274	0,084164	0,553751	0,485196	0,247240	-0,247937
PCA 10	0,623471	0,001103	0,208358	-0,175430	-0,234107	-0,441229	0,023210	0,339677	0,349548	-0,186318	-0,085247
PCA 11	-0,030117	0,033135	0,045480	0,418141	0,096799	-0,575511	-0,479514	-0,035287	-0,076126	0,419812	0,258208

**Annex 3 Table 3b**

Mixing matrix (UE27\_141\_4PCA.sta) Estimated mixing matrix computed from centered data.

	ICA 1	ICA 2	ICA 3	ICA 4					
PCA 1	-0,557621	-0,023474	-0,783854	-0,277766					
PCA 2	-0,030996	-0,089300	-0,307277	0,944764					
PCA 3	0,223245	0,952533	-0,198085	0,037411					
PCA 4	-0,802765	0,283578	0,502032	0,162215					

**Annex 3 Table 4a**

Un-mixing matrix (ue27\_14\_PCA11\_farmediaUe.sta)

Estimated un-mixing matrix computed from centered data.

	PCA 1	PCA 2	PCA 3	PCA 4	PCA 5	PCA 6	PCA 7	PCA 8	PCA 9	PCA 10	PCA 11
ICA 1	-0,219353	-0,234850	0,164472	0,112574	0,002196	0,119118	0,491175	0,307353	-0,339899	0,629164	-0,029928
ICA 2	-0,117157	0,341238	-0,398012	0,127507	-0,713631	0,246770	0,130443	0,262665	0,193153	0,000942	0,032816
ICA 3	0,518783	-0,107283	-0,294543	0,713342	0,135865	-0,165421	0,047647	-0,066953	0,163816	0,209838	0,046344
ICA 4	-0,072260	-0,079153	0,396380	0,097440	-0,362165	-0,532468	0,373357	-0,193306	0,145376	-0,176773	0,418768
ICA 5	0,227914	-0,313038	0,468618	0,117716	-0,020421	0,386713	-0,101509	0,520077	0,345629	-0,235307	0,095741
ICA 6	-0,283997	-0,416369	-0,176288	0,177836	0,012617	0,010268	0,362579	-0,105380	0,107647	-0,445511	-0,575988
ICA 7	0,372125	0,563645	0,393660	-0,041848	0,030956	0,066344	0,362106	-0,103498	0,083652	0,022522	-0,479649
ICA 8	0,250141	-0,304990	-0,220144	-0,571085	-0,037401	-0,056965	0,188344	-0,091990	0,548444	0,341349	-0,034794
ICA 9	-0,486564	0,077768	0,257713	0,282285	0,032173	0,192117	-0,255792	-0,383678	0,480808	0,350940	-0,074499
ICA 10	-0,215459	0,296712	-0,204068	0,003148	0,561443	0,189217	0,442869	0,099051	0,245436	-0,187166	0,419766
ICA 11	0,228767	-0,189213	0,020380	-0,012083	-0,160800	0,615972	0,176480	-0,581815	-0,245341	-0,085952	0,259189

**Annex 3 Table 4b**

Un-mixing matrix (UE27\_141\_4PCA.sta) Estimated un-mixing matrix computed from centered data.

	PCA 1	PCA 2	PCA 3	PCA 4					
ICA 1	-0,553641	-0,034436	0,220814	-0,798385					
ICA 2	-0,022413	-0,093359	0,955695	0,284948					
ICA 3	-0,783713	-0,308681	-0,197518	0,501378					
ICA 4	-0,275179	0,948115	0,033337	0,163809					

**Annex 3 Table 5a**

Pre-whitening matrix (ue27_14_PCA11_famediaUe.sta)											
	PCA1	PCA 2	PCA 3	PCA 4	PCA 5	PCA 6	PCA 7	PCA 8	PCA 9	PCA 10	PCA 11
ICA 1	1,002669	-0,001546	0,000317	0,000251	0,000149	-0,000026	-0,000058	0,000110	-0,000011	0,000042	-0,000095
ICA 2	-0,001546	1,000663	-0,001427	-0,001151	-0,000216	0,000048	0,000040	0,000182	0,000079	-0,000097	-0,000032
ICA 3	0,000317	-0,001427	0,995952	0,000600	-0,000056	0,000280	-0,000140	-0,000021	0,000302	0,000052	0,000014
ICA 4	0,000251	-0,001151	0,000600	1,000245	0,001788	-0,000263	-0,000417	-0,000060	-0,000016	0,000057	0,000310
ICA 5	0,000149	-0,000216	-0,000056	0,001788	1,001964	0,002154	0,001045	-0,000375	0,000058	0,000191	-0,000011
ICA 6	-0,000026	0,000048	0,000280	-0,000263	0,002154	0,997694	0,002571	0,000246	-0,000286	0,000066	-0,000038
ICA 7	-0,000058	0,000040	-0,000140	-0,000417	0,001045	0,002571	1,000496	0,001694	0,000138	0,000061	0,000022
ICA 8	0,000110	0,000182	-0,000021	-0,000060	-0,000375	0,000246	0,001694	1,000795	-0,000463	0,000701	-0,000968
ICA 9	-0,000011	0,000079	0,000302	-0,000016	0,000058	-0,000286	0,000138	-0,000463	0,995616	-0,000651	0,000240
ICA 10	0,000042	-0,000097	0,000052	0,000057	0,000191	0,000066	0,000061	0,000701	-0,000651	1,003784	0,000655
ICA 11	-0,000095	-0,000032	0,000014	0,000310	-0,000011	-0,000038	0,000022	-0,000968	0,000240	0,000655	1,000310

**Annex 3 Table 5b**

Pre-whitening matrix (UE27_141_4PCA.sta)				
	PCA 1	PCA 2	PCA 3	PCA 4
ICA 1	0,998472	0,001088	0,000978	-0,001193
ICA 2	0,001088	1,002049	-0,002123	0,000719
ICA 3	0,000978	-0,002123	1,001114	0,001231
ICA 4	-0,001193	0,000719	0,001231	0,998410

**Annex 3 Table 6a**

Covariance matrix (ue27_14_PCA11_famediaUe.sta) The covariance matrix of centered and standardized (if specified) data.											
	PCA1	PCA 2	PCA 3	PCA 4	PCA 5	PCA 6	PCA 7	PCA 8	PCA 9	PCA 10	PCA 11
PCA1	1,000000	0,006647	-0,000512	-0,000001	0,000000	0,000000	0,000000	0,000000	0,000000	0,000000	0,000000
PCA 2	0,006647	1,000000	-0,003913	-0,000006	0,000000	0,000000	0,000000	0,000000	0,000000	0,000000	0,000000
PCA 3	-0,000512	-0,003913	1,000000	0,001145	-0,000026	-0,000016	-0,000021	0,000009	-0,000003	0,000001	0,000000
PCA 4	-0,000001	-0,000006	0,001145	1,000000	-0,004613	-0,000546	0,000005	0,000000	0,000000	0,000000	0,000000
PCA 5	0,000000	0,000000	-0,000026	-0,004613	1,000000	-0,005489	0,000087	0,000000	-0,000002	0,000000	0,000000
PCA 6	0,000000	0,000000	-0,000016	-0,000546	-0,005489	1,000000	-0,005572	0,000067	0,000270	-0,000021	-0,000015
PCA 7	0,000000	0,000000	-0,000021	0,000005	0,000087	-0,005572	1,000000	0,004960	0,000017	0,000021	-0,000003
PCA 8	0,000000	0,000000	0,000009	0,000000	0,000000	0,000067	0,004960	1,000000	0,003842	0,001970	-0,000325
PCA 9	0,000000	0,000000	-0,000003	0,000000	-0,000002	0,000270	0,000017	0,003842	1,000000	0,006191	-0,000082
PCA 10	0,000000	0,000000	0,000001	0,000000	0,000000	-0,000021	0,000021	0,001970	0,006191	1,000000	-0,004807
PCA 11	0,000000	0,000000	0,000000	0,000000	0,000000	-0,000015	-0,000003	-0,000325	-0,000082	-0,004807	1,000000

**Annex 3 Table 6b**

Covariance matrix (UE27_141_4PCA.sta) The covariance matrix of centered and standardized (if specified) data.				
	PCA 1	PCA 2	PCA 3	PCA 4
PCA 1	1,000000	0,006647	-0,000512	-0,000001
PCA 2	0,006647	1,000000	-0,003913	-0,000006
PCA 3	-0,000512	-0,003913	1,000000	0,001145
PCA 4	-0,000001	-0,000006	0,001145	1,000000

Covariance measure dependence of x on y, non normalised, where x, y are independent.

Eigenvalues are equal to variance of projections along corresponding eigenvector ( $\sigma^2 = \xi$ ) Eigenvectors of a symmetric matrix are orthogonal, normalized basis vectors for a coordinate system. Total variance in data is given by sum of eigenvalues.<sup>33</sup>

$$C = \begin{bmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{bmatrix} = \begin{bmatrix} \xi_1 & 0 \\ 0 & \xi_2 \end{bmatrix}$$

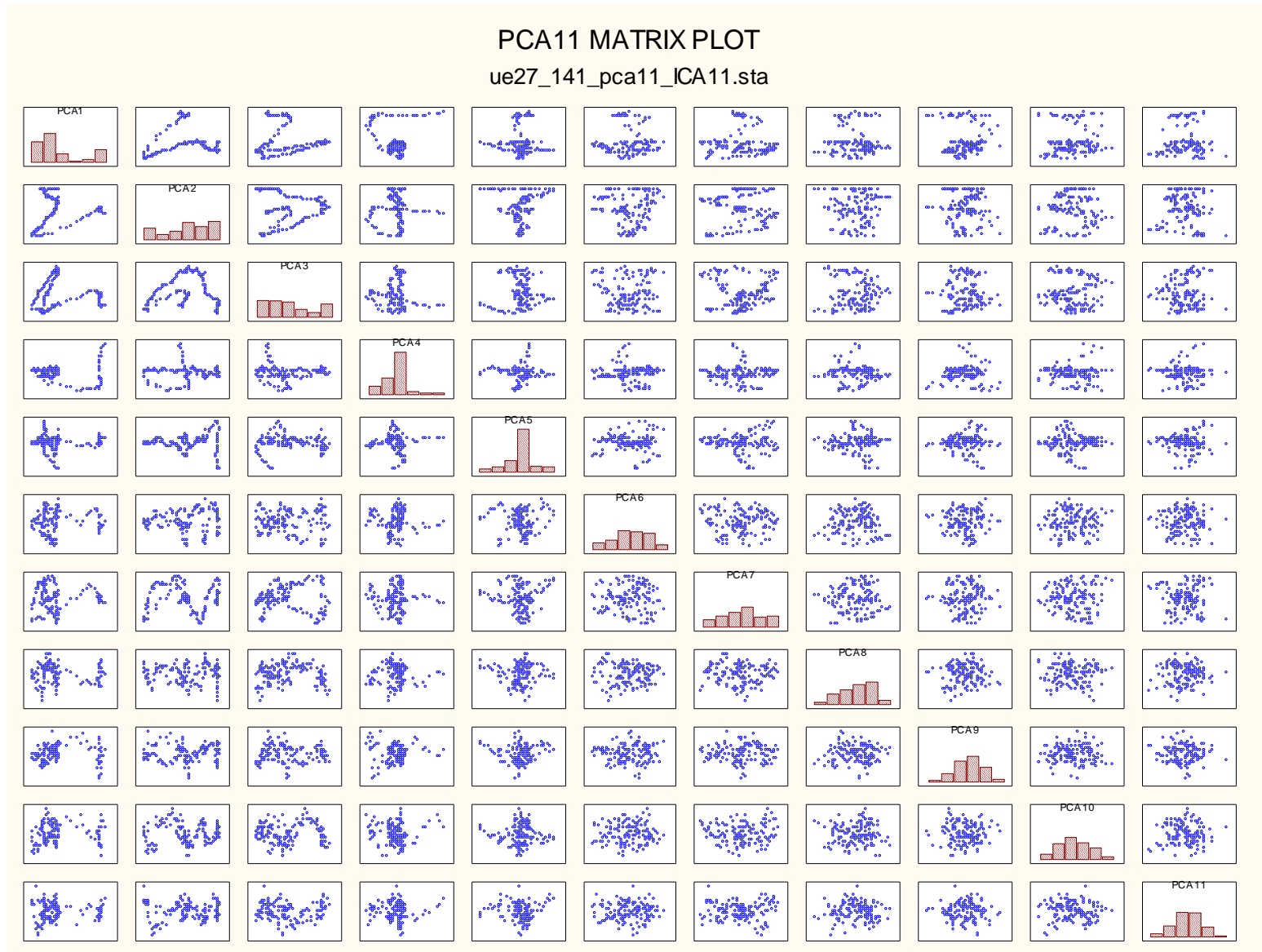
Annex 3\_Table 7a

Descriptive statistics of separated source data (ue27_14_PCA11_famediaUe.sta) Number of components: 11					Descriptive statistics for variables (ue27_14_PCA11_famediaUe.sta) Number of valid cases: 141				
	Mean	Variance	Minimum	Maximum		Mean	Variance	Minimum	Maximum
ICA 1	0,000000	0,999385	-3,97801	1,934869	PCA1	0,000000	10,23803	-3,97392	7,032930
ICA 2	0,000000	0,998537	-0,92013	4,477845	PCA 2	0,000000	8,58839	-5,57076	3,936121
ICA 3	0,000000	0,999532	-1,45835	3,665043	PCA 3	0,000000	5,22154	-3,32295	4,364150
ICA 4	0,000000	0,995466	-2,79739	1,896845	PCA 4	0,000000	1,01539	-2,31210	3,822250
ICA 5	0,000000	0,996458	-4,03926	1,499888	PCA 5	0,000000	0,54612	-2,22974	1,838798
ICA 6	0,000000	1,005686	-3,48735	1,948706	PCA 6	0,000000	0,38975	-1,40335	1,334375
ICA 7	0,000000	0,996886	-2,06508	3,567277	PCA 7	0,000000	0,26260	-1,06926	0,974273
ICA 8	0,000000	1,005739	-1,72374	3,954616	PCA 8	0,000000	0,19164	-1,19199	0,961533
ICA 9	0,000000	1,001332	-3,18592	2,634494	PCA 9	0,000000	0,12364	-1,04479	0,921608
ICA 10	0,000000	1,000690	-3,99528	1,696116	PCA 10	0,000000	0,09917	-0,70351	0,799567
ICA 11	0,000000	1,000859	-1,34836	3,559213	PCA 11	0,000000	0,07257	-0,74287	0,817664

Descriptive statistics of separated source data (UE27_141_4PCA.sta) Number of components: 4					Descriptive statistics for variables (UE27_141_4PCA.sta) Number of valid cases: 141				
	Mean	Variance	Minimum	Maximum		Mean	Variance	Minimum	Maximum
ICA 1	0,000000	1,001693	-1,83510	1,575418	PCA 1	0,000000	10,23803	-3,97392	7,032930
ICA 2	0,000000	0,998915	-2,37548	2,055181	PCA 2	0,000000	8,58839	-5,57076	3,936121
ICA 3	0,000000	1,000186	-1,29439	4,432552	PCA 3	0,000000	5,22154	-3,32295	4,364150
ICA 4	0,000000	0,999267	-1,73535	1,388644	PCA 4	0,000000	1,01539	-2,31210	3,822250

Annex 3\_Table 7b

<sup>33</sup> \*\*\*, Why is dimensionality reduction useful?, <http://rieke-server.physiol.washington.edu/People/Fred/Classes/545/PCA2.pdf>



### ICA11 MATRIX PLOT ue27\_141\_pca11\_ICA11.sta

