# The Flexible Accelerator Model and the 'Regionalization' of Capital Stock Estimates

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**Abstract**
*This paper suggests a model of obtaining estimates of capital stock based on the theory of 'flexible accelerator'. However, this represents a rather 'indirect' method independently for each year and each region. Clearly this is an unrealistic condition, especially for regional economies characterized by mutual spatial dependence. To add an extra injection of realism, we illustrate how a national model of capital stock (the stock –flow model) can effectively be 'regionalized'.*

**Keywords:** capital stock; flexible accelerator; spatial analysis, stock-flow

## 1. Introduction

The analysis of regional economic growth has always been handicapped by the almost unavailability of data on regional capital stock. In the absence of reliable time series, it is difficult to assess fully the contribution of investment activities (private or public) into the growth performance of regional economies. Actual capital stock data are seldom available and often inadequate, especially at the regional level of analysis. To try to sort this out, several studies (e.g. Harris, 1983; Hulten and Schwab, 1984; Gertler, 1986; Anderson and Rigby, 1989), estimate the capital stock using data on investment. In this context, a useful approach is provided by acceleration theory in its 'flexible' version, i.e. the relation between the investment activity and output (Junankar, 1973). Although this is an 'indirect' method, nevertheless, it allows the researcher to obtain estimates of the annual amount of the capital stock and depreciation independently for each year and each region. A useful extension of this method would be the incorporation of spatial spillovers effects. How this might be effects is tentatively illustrated in the context of the stock-flow model. The remainder of this paper is organised as follows. Section 2 outlines the simple version of the 'flexible' accelerator model while an attempt to 'regionalise' the stock-flow is reported in section 3. Section 4 concludes.

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The findings, interpretations and conclusions are entirely those of the authors and, do not necessarily represent the official position, policies or views of the Ministry of Rural Development & Foods and/or the Greek Government.

## 2. The 'Flexible Accelerator' model

One of the most common methods of estimating capital stock is the so-called *perpetual inventory* method[1]. In essence, this technique requires a time series of deflated values of capital investment as input data. These data are calculated by dividing the current value of investment ($\tilde{I}_t$) in each time period ($t$) by a capital goods index ($P_c$):

$$I_t = \frac{\tilde{I}_t}{P_c} \tag{1}$$

where $I_t$ is the deflated value of investment in period $t$.

The expression in equation (1) adjusts for changes in the purchasing power of the monetary value of investment. The measurement of capital in monetary terms is justified in terms of the microeconomic theory, in which the price a firm is willing to pay for a capital good is equal to the value of the discounted flow of revenues it is expected to produce. Consequently, the productive capacity, measured in monetary terms, of two capital goods of the same vintage with equal prices must be equal. However, this is not necessarily true, given that it is possible technology and preferences to change through time. According to the perpetual inventory method, depreciation is estimated using a single parameter ($\mu$), which is the assumed service lifetime of capital goods (Anderson and Rigby, 1989). A capital good is assumed to be completely withdrawn from the capital stock after $\mu$ years. Hence, the simplest assumption governing depreciation is that a capital good remains in full service until is withdrawn suddenly at the end of its lifetime service[2]. Under this assumption capital stock at the beginning of period $t$ is:

$$K_t = \sum_{v=t-\mu}^{t-1} I_v \tag{2}$$

An alternative way is to assume that following its installation, an equal proportion of the services of capital is withdrawn in each of the $\mu$ periods (straight line depreciation). Therefore,

$$K_t = \left(1 - \frac{1}{\lambda}\right) K_{t-1} + I_{t-1} \tag{3}$$

Equation (3) incorporates the fact that capital goods may be removed from production at any time after they installed. In this light the parameter $\mu$ reflects the maximum service lifetime. However, most important empirical problem is to estimate the parameter $\mu$. In the relevant literature, $\mu$ is estimated based on engineering surveys, which assess the average service lifetimes of capital goods.

Based on the notion of 'optimal' capital stock, the model of 'flexible accelerator' involves a relation between capital stock ($K$) and output ($Y$):

$$K^*_{i,t} = \alpha Y_{i,t} \text{ and } K^*_{i,t-1} = \alpha Y_{i,t-1} \tag{4}$$

where $a$ is a parameter and the subscript $i$ refers to a specific region.

The adjustment function is expressed as follows:

$$K_{i,t} - K_{i,t-1} = (1-\lambda)(K^*_{i,t} - K_{i,t-1}), \text{ with } 0 < \lambda < 1 \tag{5}$$

The replacement or the depreciation of capital is given by the following equation:

$$D_{i,t} = \delta K_{i,t-1}, \text{ with } 0 < \delta < 1 \tag{6}$$

Capital stock in any period $t$, is the sum of the existing capital stock plus the new additions by net investment in current period:

$$K_{i,t} = K_{i,t-1} + I_{i,t} - D_{i,t} \tag{7}$$

where $I_{i,t}$ is the level of gross investment.

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[1] Chenery (1952), Duesenberry (1958), Eisner (1960) and Patterson and Schott (1978) are obvious references for a more detailed analysis.

[2] Solow (1962) called this assumption as 'one hoss hay'.

Solving equation (5) for $K_{i,t}$ yields

$$K_{i,t} = (1-\lambda)K^{*}_{i,t} + \lambda K_{i,t-1} \tag{8}$$

Inserting equation (6) into equation (7) yields an expression for gross investment:

$$I_{i,t} = K_{i,t} - (1-\delta)K_{i,t-1} \tag{9}$$

Using equation (8), equation (9) is written as follows:

$$I_{i,t} = (1-\lambda)K^{*}_{i,t} - (1-\lambda-\delta)K_{i,t-1} \tag{10}$$

Lagging equations (5) and (9) by one period yields

$$K_{i,t-1} = (1-\lambda)K^{*}_{i,t-1} + \lambda K_{i,t-2} \tag{11}$$

$$K_{i,t-2} = \frac{(K_{i,t-1} - I_{i,t-1})}{(1-\delta)} \tag{12}$$

Therefore,

$$I_{i,t} = \alpha(1-\lambda)Y_{i,t} - \alpha(1-\lambda)(1-\delta)Y_{i,t-1} + \lambda I_{i,t-1} \tag{13}$$

Equation (13) has the advantage of providing an estimate of the capital stock in each region using data for output and investment[3].

Expressing equation (13) in terms of a regression equation yields:

$$I_{i,t} = c + b_1 Y_{i,t} + b_2 Y_{i,t-1} + b_3 I_{i,t-1} \tag{14}$$

where $b_1 = \alpha(1-\lambda), b_2 = -\alpha(1-\lambda)(1-\delta), b_3 = \lambda$ and $c$ is the constant term of the regression.

Quantitatively the most important fact is that the parameters $b_1, b_2$ and $b_3$ allow to estimate the values of $a$ and $\delta$. Therefore,

$$\alpha = \frac{\hat{b}_1}{1-\hat{b}_3} \text{ and } \delta = \frac{\hat{b}_1 + \hat{b}_2}{\hat{b}_1} \tag{15}$$

Using the parameters $a$, $\delta$ and $\lambda$ is possible to obtain an estimate for the capital stock for the period $t-1$. Solving equation (10) for $K_{i,t-1}$ and rearranging yields

$$K_{i,t-1} = \frac{(1-\lambda)}{(1-\lambda-\delta)}K^{*}_{i,t} - \frac{1}{(1-\lambda-\delta)}I_{i,t} \tag{16}$$

The capital stock of period in $t-1$ can be estimated as follows[4]:

$$K_{i,t-1} = \frac{\alpha(1-\lambda)}{(1-\lambda-\delta)}Y_{i,t} - \frac{1}{(1-\lambda-\delta)}I_{i,t} \tag{17}$$

Implementing equations (7) and (6) and the estimated capital stock in period $t-1$, capital stock and the amount of depreciation in each period are possible to be determined.

The analysis thus far, although refers to regions, it does not account for spatial interaction; that is to say how investment in one region impacts on the neighbouring regions. In principle, the 'flexible' accelerator model could be adapted to a regional framework for estimating capital stock. It has been argued, particularly in the case of regional economies, that the location of a region within a system of regional economies is a unique characteristic, and in the same way as other structural characteristics, has the potential to impact on growth and development. Regions are not dimensionless points but vital functional parts of an inter-dependent system of regional economies. This economic inter-dependence is partly a function of spatial inter-dependence. We now turn to how incorporating an explicitly spatial approach.

[3] Wallis (1973) and Helliwell (1976) used this indirect method of estimating the stock of capital of aggregate economy. Katos (1978) applied a similar approach to estimate the capital stock for the entire Greek economy using data covering the period 1948-1972.

[4] If $\lambda + \delta = 1$ brings indeterminacy in the model.

### 3. Regionalizing a national capital stock model.

As noted above, the flexible accelerator model posits a relationship between investment and output. It is based on the assumption of a stock adjustment process between a firms 'desired' level of capital stock and its actual level. The rate of change of actual capital stock will be proportional to the difference between the desired and actual stock (Lucas, 1969). As such, it can be used as a starting point for estimating capital stocks. Garofalo and Malhotra (1973) have used it to estimate the impact of changing input prices on patterns of investment and capital stock in US manufacturing investment in the 1970's.
'Regionalising' such estimates however can be challenging. The basic expression for investment in region $j$ is;

$$I_j = a + \beta \delta Y_j + u_j \tag{18}$$

The 'flexibility' is due to the fact that $\beta$ is not fixed. Change in output drives investment. Output itself depends on capital stock and in order to keep boosting the former, investment is required in the latter. A key issue is that there is no real theory that posits investment as dependent on change in output. Thus the flexible accelerator is more of an ad hoc Keynesian behavioural construct than a fully fledged model. Additionally there is nothing inherently 'regional' in equation (18) as it is simply a national relation applied to a given region.
However, it is reasonable to claim that estimates of regional capital stock from investment and output data may not just be a case of regionally apportioning national estimates. This is because regional investment is unlikely to be independent. Just as regional models of housing construction and prices are not national models writ locally, the same can be said for other forms of regional capital stock. Additionally we can posit the existence of regional capital stock spillover effects as the stock of capital in one region is affected by the level or amount of stock in a neighbouring region (spatial lag effect)[5].

A different route to take may be by regionally invoking the 'stock-flow' asset pricing model that has been used in housing research to the issue of estimating non-residential capital stock. This model has a dynamic adjustment process that regulates prices with stocks of assets so that prices are determined as an asset while the flow of this asset is determined by investment which in turn is contingent of price levels (see for example, Smith, 1969; Bar-Nathan et al, 1998). In the stock-flow model, prices are 'weakly' exogenous because they are determined in the asset market, and capital stock is large relative to the flow of new capital construction (Topel and Rosen, 1988). In addition, the fact that capital stock does not become obsolete quickly makes for the assumption of a competitive investment environment.
This stock-flow model can be given a spatial expression that takes spillovers into account. Given two regions (A and B) in which the level of firms ($Q$) is fixed, $Q_{At}$, where $Q_{At}$ and $Q_{Bt}$ are naturally positive. The firm choosing to locate in A is determined through the following condition:

$$Q_{At} = \varphi_0 - \varphi_1 P_{At} + \varphi_2 P_{Bt} \tag{19}$$

where $P_A$ denotes price of capital stock in region A. The coefficients $\varphi_1$ and $\varphi_2$ reflect regional locational preferences across firms and imply that regions are imperfect locational substitutes for each other. (If they are perfect substitutes $\varphi_1 = \varphi_2 = \infty$. At the other extreme, if there is no substitution at all $\varphi_1 = \varphi_2 = 0$).
We assume that the capital cost for constructing stock (building materials, labour) does not differ across regions. These inputs are tradable and contractors will choose to build where it is more profitable. However, there is in general imperfect substitution between building in A and B because contractors have regional preferences too, or their expertise is region-specific. Given everything else, contractors therefore build more stock in A if they can sell for higher prices Construction of capital stock, denoted by $C$, is determined as follows in regions A and B:

$$C_{At} = \eta_{A0} + \eta_{A1} P_{At} - \eta_{A2} P_{Bt} \tag{20}$$

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[5] For a demand side example of regional spillover effects in a housing context, see Beenstock and Felsenstein (2010).

$$C_{Bt} = \eta_{B0} + \eta_{B1} P_{Bt} - \eta_{B2} P_{At} \quad (21)$$

where $\eta_{A0}$ and $\eta_{B0}$ express productivity in construction in regions A and B respectively. Capital stock at the beginning of period t in the two regions is defined as:

$S_{jt} = S_{jt-1} + C_{jt-1} - d_{jt-1}$ , where $j = A, B$ and $d$ denotes demolitions (22)

The market is in equilibrium when $Q_{jt} = S_{jt}$.

The model for stock prices can be solved under the simplifying assumption that $d = \delta S - 1$, where $\delta$ is a common demolition rate. Stock prices are dynamically and spatially correlated according to the model so that prices in region A are related to lagged prices in regions A and B, as well as current prices in region B:

$$P_{At} = \tfrac{1}{\varphi_1}\left[\varphi_0 - \eta_{A0} - \eta_{A1} P_{At-1} + \varphi_2 P_{Bt} + \eta_{A2} P_{Bt-1} - (1-\delta) S_{At-1}\right] \quad (23)$$

Current prices in region A vary inversely with the local stock and construction productivity ($\eta_{A0}$), and vary directly with the autonomous demand to locate in A ($\varphi_0$).

A crucial assumption in equations (20) and (21) is that apart from substitution effects induced by $\eta_{A2}$ and $\eta_{B2}$ the regions are otherwise independent in terms of supply. A further source of dependence is induced by specifying spatial lags in equations (20) and (21). Thus in equation (20) and (21) we add $\lambda_A \eta_{Bt}$, where $\lambda$ denotes the spatial lag coefficient. If $\lambda > 0$, regional construction is complementary and induces mutual crowding-in. Thus, public policy to induce regional capital stock attracts further capital investment. If $\lambda < 0$, public sector regional construction induces mutual crowding-out of any further capital investment.

The operational model for estimating equations of regional capital stock including spillover effects across regions would then be:

$$B_{it} = \alpha_i + \eta(P_{it} - CN_{it}) + \phi(P^*_{it} - CN^*_{it}) + \gamma(\overline{P}_t - CN_t) + \lambda B^*_{it} + \mu Z_{it} + \pi Z^*_{it} + u_{it} \quad (24)$$

where all variables are expressed as logarithms, $CN$ denotes building costs and * denote a spatial lag, e.g.:

$$B^*_i = \sum_{j \neq i}^{N} w_{ij} B_i \quad (25)$$

where $w_{ij}$ denote exogenous spatial weights row-summed to unity. The main hypotheses are that capital stock construction varies directly with profitability, hence $\eta > 0$, and it varies directly with public sector intervention, hence $\mu > 0$. Equation (24) includes three spatial effects. First, if profitability increases among the neighbours of region $i$ contractors will engage in spatial substitution, hence $\phi < 0$. Secondly, if regional incentives received by the neighbours of region $i$ induce spatial substitution in construction, $\pi$ will be negative. However, if construction in region $i$ and its neighbours are complementary $\pi$ may be positive. Third, if there are positive spatial spillovers in construction $\lambda$ will be positive.

Apart from a spatial effect on profitability ($\phi$), equation (24) includes a national effect ($\gamma$). If local and neighbouring profitability are given, an increase in national profitability might affect local capital stock construction in two ways. First, substitution may take place beyond neighbouring regions, which would make $\gamma$ negative. Secondly, an increase in national profitability has a positive effect on national construction. If national and local construction are complements then $\gamma$ may be positive.

## 4. Conclusion

While physical capital in manufacturing is a crucial variable in regional economic analysis, little attention has been given to developing accurate methods of estimating time series of regional capital stock. This paper is devoted to an inspection of methods to obtain estimates of capital stock, based on the 'flexible' accelerator and stock flow models. Of particular relevance to capital accumulation across regions are the effects of spatial interaction which are discussed in this paper. Such estimates may be useful in a variety of regional economic analyses. For example, for estimating production-function type models in testing macroeconomic theories of regional growth, obtaining regional capital/labour ratios, technical efficiency, capital ages, etc. However, this is ultimately an empirical issue. Indeed, there is a little pretence that the foregoing analysis provides an exhaustive account of all the factors that affect the capital accumulation across regions, but this work does provide an alternative approach to the estimation of capital stock, and suggests possible avenues for future research in different contexts and examining different factors that shape the pattern of capital accumulation.

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